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Essays on Exchange Rates: Central Banks' Interventions, Effects on Gold Mining Activity, and Anticipating Market Risk

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ABSTRACT

The essays in this work explore exchange rate related topics of different areas of study. We start exploring the notion of central banks' interventions in the exchange market. Particularly, we are interested in testing the well known Fear of Floating hypothesis. Then, we change gears to study the effects of a volatile exchange rate on the extraction of a natural resource. Gold mining activity around the world is our subject of interest. Last, we contribute to the risk analysis literature by adding a new parameter (market trend) into the volatility-weighted historical simulation approach to better anticipate market risk for different groups of assets including exchange rates.

The first essay uses a new approach to identify the exchange policies of central banks around the world. We focus directly on the lagged relationship between changes in international reserves and changes in the exchange rate. The evidence does not support the fear of floating hypothesis. Central banks have accumulated reserves after exchange rate depreciations as well as after appreciations. Rather than constantly intervening in the exchange market, our evidence indicates that central banks appear to be accumulating reserves as insurance against future financial crises.

The second essay poses and tests a dynamic optimal control production/location model of a representative global gold mining company. The firm must decide where, when, and by how much to expand gold production in each country given each country's in-ground gold reserves. First, we assume that both gold prices and exchange rates are

known in advance with certainty. The optimal extraction path in this case balances revenues with production costs and the "user cost" of depleting the known reserve, in common currency terms, in each country. The more realistic model formalizes the real-world uncertainty about gold prices and exchange rate risk. In this model, exchange rates follow a stochastic differential process with a direct dependence on time and space. The results indicate that prices of gold and country exchange rates have been significant determinants of the location and intensity of gold mining activity around the world.

The third essay aims at verifying empirically the applicability of adding a new parameter (market trend) into the volatility-weighted historical simulation approach to anticipate market risk (VaR and CVaR) for different groups of assets (exchange rates, financials, metals, and energies). Volatilities are forecasted using EWMA and GARCH models. The results show that adding the market trend parameter can improve VaR estimates, especially for extreme VaRs (99% and 1%). We also observe that overall using a well calibrated EWMA model to forecast volatility provides better VaR estimates for the volatility-weighted historical simulation approach than using the GARCH model. In terms of CVaR estimation the results are ambiguous. There is no prevalence of one model over the other. Also, the contribution of the trend parameter to these models in estimating CVaRs is not evident.

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1. Introduction

Since its emergence the fear of floating hypothesis has been central in the toolkit for deducing the actual exchange rate regimes of developing countries. A large body of research sustaining the fear of floating hypothesis documented that developing countries have not only less volatile nominal exchange rates but also significantly more volatile international reserves.[1] Is this evidence or circumstantial?

Rationales for fear of floating are various. De Nicoló, Honohan, and Ize (2003), for example, associate the fear of floating behavior with the existence of large un-hedged foreign currency-denominated liabilities in the economy.[2] Hausmann, Panizza, and Stein (2001) find that the higher the degree of passthrough from exchange rate to prices, the lower the degree of exchange rate flexibility. Loss of access to international capital markets due to a lack of credibility is another explanation found for the fear of floating behavior (Calvo and Reinhart (2000)). Further, Calvo and Reinhart (2002) believe in a combination of lack of credibility, high pass-through from exchange rates to prices, and inflation targeting as an additional reason why fear of floating may arise.

On the other hand, central banks have to consider that interventions on exchange rate depreciations are costly in the sense that they consume reserves. The cost of having

[1] A partial list includes Calvo and Reinhart (2002), Hausmann, Panizza, and Stein (2001), and Levy-Yeyati and Sturzenegger (2002).

[2] Calvo, Izquierdo, and Talvi (2002), Calvo, Izquierdo, and Mejía (2004), and Honig (2005) also cite domestic liability dollarization, including both dollar deposits and dollar credit, as another source to create currency mismatches.

low levels of reserves during an eventual financial crisis might exceed by far the costs generated by a more volatile currency. Therefore, central banks have to choose where to rest their monetary policy in the exchange rate flexibility spectrum given each country's circumstances at a given point in time.

By considering the sequence of events characterizing an intervention, the first essay uses a new approach to identify the exchange policies of central banks around the world. We estimate a parameter that measures intervention with respect to either direction of the change in an exchange rate in an alternative way that i) respects the sequence of events that characterize an intervention; ii) incorporates the probability of an intervention occur; and iii) takes into account the expected amount used to intervene. We also evaluate the correlation between international reserves and exchange rates separately for the cases of appreciation and of depreciation. The evidence does not support the fear of floating hypothesis. Central banks have accumulated reserves after exchange rate depreciations as well as after appreciations. Rather than constantly intervening in the exchange market, our evidence indicates that central banks appear to be accumulating reserves as insurance against future financial crises. In sum, it appears that developing countries are willing to let their currency to float, but with a "life jacket."

Volatile exchange rates can have a broad effect on an economy depending on its exposure to the exchange rate risk. One particular sector that is largely affected by exchange rate risk is the international tradeable goods sector. Gold is obviously an international tradeable good. In contrast with market-oriented service sector activity or other resource-oriented activities like farming, however, mining sector activity around the world is extremely volatile. Two volatile financial variables: the world price of gold and local currency exchange rates, may explain why mining is such a volatile industry.

The second essay poses and tests a dynamic optimal control production/location model of a representative global gold mining company. It elaborates the basic canonical model of optimal resource exploitation of Hotelling (1931), as formalized by Pindyck (1978, 1980) most recently, while extending it to consider alternative locations for extractive activity. The firm must decide where, when, and by how much to expand gold production in each country given each country's in-ground gold reserves. In the first model, both gold prices and exchange rates are assumed known in advance with certainty. The optimal extraction path in this case balances revenues with production costs and the "user cost" of depleting the known reserve, in common currency terms, in each country. The more realistic model formalizes the real-world uncertainty about gold prices and exchange rate risk. In this model, exchange rates follow a stochastic differential process with a direct dependence on time and space. Empirical validity tests on data about the global gold sector since 1970 indicate that prices of gold and country exchange rates have been significant determinants of the location and intensity of gold mining activity around the world.

One technique widely used to monitor market risk on exchange rates is the Value-at-Risk (VaR).[3] Given a time horizon and confidence level, VaR estimates the maximum loss that can possibly occur. It is essentially a static model (one-period model) in which positions remain unchanged over the risk horizon. The appeal of the VaR measure relies on the fact that it aggregates several components of risk into one single number. However, an important shortcoming of VaR is that it disregards any loss beyond its critical value. This problem, called the tail risk, was address by Artzner et al. (1997) and Embrechts et al. (1998). Being aware of this fact is so critical that, as shown by Yamai and Yoshiba (2005), in certain real-world cases investors may take wrong decisions based on VaR. To circumvent the main weakness of VaR, Rockafellar and Uryasev (2000, 2002) suggest the Conditional Value-at-Risk, or CVaR,[4] which takes into account losses outside the VaR quantile. Supported by the extreme value theory (EVT), CVaR represents the conditional expectation of loss (mean loss) that is beyond the VaR level. It basically indicates what we can expect to lose given the loss is beyond VaR. As pointed by Dowd (2002), CVaR has the many attractions of the VaR measure and, further, it has also some additional advantages: it tells the analyst what to expect in bad states (tail events); it does not discourage risk diversification as VaR sometimes does; and CVaR estimates are less prone to sampling error than VaR. Although CVaR has many

[3] In the mid-90s, the bank JP Morgan developed a standard risk measure for financial risk management, called Value-at-risk. The idea was to create a general measure of economic loss that could incorporate the diverse risk across positions and aggregate them on a portfolio basis.
[4] This risk measure is also called in the literature as expected tail loss, expected shortfall, tail VaR, tail conditional expectation and worst conditional expectation.

advantages over VaR, the truth is that these risk measure techniques complement each other. VaR informs investors about ordinary loses while CVaR focus on unusual loses. Therefore, these methods are not exclusives but rather should be used together to provide a more complete risk estimate.

The third essay aims at verifying empirically the applicability of adding a new parameter (market trend) into the volatility-weighted historical simulation approach (Hull and White (1998)) to anticipate market risk (VaR and CVaR) for different groups of assets (exchange rates, financials, metals, and energies). Volatilities are forecasted using EWMA and GARCH models. The results show that adding the market trend parameter can improve VaR estimates, especially for extreme VaRs (99% and 1%). We also observe that overall using a well calibrated EWMA model to forecast volatility provides better VaR estimates for the volatility-weighted historical simulation approach than using the GARCH model. In terms of CVaR estimation the results are ambiguous. There is no prevalence of one model over the other. Also, the contribution of the trend parameter to these models in estimating CVaRs is not evident.

2. Fear of Floating: Phantom or Fact?

2.1. Introduction

Since its introduction by Calvo and Reinhart (2002), the fear of floating hypothesis has occupied a central place in the debate on exchange rate regimes. Fear of floating refers to the use of market instruments – international reserves and/or interest rates – by central banks to reduce the volatility of the exchange rate. Control of exchange rate volatility is achieved mainly by (1) selling (buying) international reserves causing a direct increase in supply (demand) of foreign currency when an excess demand (supply) is in place; or (2) increasing (decreasing) interest rates by increasing (decreasing) domestic money supply which can indirectly induce supply (demand) of foreign currency.

In their pioneer work, Calvo and Reinhart observed the frequency distribution of monthly percent changes in the exchange rate and foreign exchange reserves to test whether countries claiming as to be floaters are in fact floating. The idea is simple. In fixed exchange rate regimes, the probability that the percent change in exchange rate falls within an (arbitrary) band is greatest, while the probability that the percent change in foreign reserves is within an arbitrary band is lowest. In contrast, in free floating regimes these probabilities are reversed. Anything in the middle characterizes limited flexibility or managed floating arrangements. Calvo and Reinhart assessed the extent of intervention by analyzing the behavior of exchange rates and international reserves relative to bands of ±2.5 percent. Classifying countries with a floating regime tradition as benchmarks,[5] the authors concluded that compared to the benchmark countries, most countries intervene more in the exchange market. This is the basic evidence of widespread fear of floating currency regimes.

This paper tests the fear of floating hypothesis using a different approach. We focus directly on the lagged relationships between changes in international reserves and changes in the exchange rate. We estimate a parameter that measures intervention with respect to either direction of the change in an exchange rate in an alternative way that i) respects the sequence of events that characterize an intervention; ii) incorporates the probability of an intervention occur; and iii) takes into account the expected amount used to intervene. We also evaluate the correlation between international reserves and exchange rates separately for the cases of appreciation and of depreciation. The null hypothesis is that fear of floating is a phantom.

The results show that the idea that many countries' foreign reserves are volatile because their central banks are constantly intervening to stabilize their country's exchange rate may be misleading. Increasing reserves prevails on either side of the movement in the exchange rate. There is widespread tendency to increase reserves even

[5] Among developed countries, the United States is the benchmark. For emerging economies, Canada is the benchmark.

after a depreciation of the domestic currency rather than to intervene, especially when an increase in exchange rate volatility has occurred. Central banks are apparently more concerned with accumulating reserves and, therefore, buying insurance against an eventual financial crisis (sudden stops, for example), rather than to stabilize their currency. Perhaps central bankers believe that is better to be prepared for a crisis rather than attempt to avoid it unilaterally by using up their reserves to hold their exchange rate stable.

This essay is structured as follows. Section 2 presents a brief discussion about the rationale for intervening or not in the exchange market. Section 3 derives the new parameter developed for this study to measure intervention. Section 4 discusses the data set and the empirical results. Finally, section 5 summarizes the findings.

2.2. Intervene or Not to Intervene?

A contemporary debate in monetary policy involves the direction in which central banks are moving in the exchange rate flexibility spectrum. Some researchers ascribe to a bipolar (corner solution) tendency.[6] Others, following the work of Calvo and Reinhart

[6] The bipolar hypothesis states that countries move towards to the edge of the exchange rate arrangement flexibility spectrum tending to either the most rigid forms of pegged regimes (such as currency board arrangements) or total independence of the monetary policy relative to the exchange rate policy as the independent floating regime. For discussions about the bipolar view see Obstfeld and Rogoff (1995), Eichengreen and Masson (1998), Peterson, Goldstein, and Hills (1999), and Fischer (2001).

(2002), claim that central banks are tending towards the centre of the spectrum, given the spread fear of floating among countries around the world.

Much research sustaining the fear of floating hypothesis shows that developing markets have not only less volatile nominal exchange rates but also more volatile international reserves and domestic interest rates, compared to developed markets (e.g., Calvo and Reinhart (2002), Hausmann, Panizza, and Stein (2001), Levy-Yeyati and Sturzenegger (2002)). Assuming that developing countries are vulnerable to currency crises, these authors conclude that the reason behind the unexpected observed low volatility of the exchange rate in these countries is that they follow a policy of intervention to dampen exchange rate fluctuations.

Rationales for fear of floating are various. De Nicoló, Honohan, and Ize (2003), for example, associate the fear of floating behavior with the existence of large un-hedged foreign currency-denominated liabilities in the economy. This persuasive argument is as follows. If the banks in the economy are not fully hedged against a depreciation of the domestic currency, and if liabilities denominated in each foreign currency are greater than assets in the each currency (called the *currency mismatch*[7]), a large depreciation can lead to significant reductions in net worth, ultimately driving banks into bankruptcy. This process can lead to a sharp decline in the available credit in the economy, affecting investments in production and causing a contraction in output, ultimately bringing the

[7] This is a typical scenario observed in many developing countries where the financial system is not well equipped with instruments for hedging and firms and banks borrow extensively from abroad (see Burnside, Eichenbaum, and Rebelo (1999) and Hausmann, Panizza, and Stein (2001) for empirical evidences).

economy to a recession. Furthermore, Calvo, Izquierdo, and Talvi (2002), Calvo, Izquierdo, and Mejía (2004), and Honig (2005) conclude that domestic liability dollarization, including both dollar deposits and dollar credit, is even more likely to create currency mismatches. This is because firms that borrow domestically in foreign currency usually earn revenue in domestic currency (especially when the economy is relatively closed) and, therefore, are more likely to default a loan when a large depreciation occurs. Firms that borrow from abroad are usually in the export sectors and are more likely to earn revenues in dollars as well.

Another reason of why fear of floating may arise is given by Hausmann, Panizza, and Stein (2001). They find that the higher the degree of passthrough from exchange rate to prices, the lower the degree of exchange rate flexibility. The reason for this is that a positive change in the exchange rate has a direct and, although more limited, indirect effect on prices. Depreciations of the domestic currency cause an immediate upward pressure on prices of tradeable goods in the economy. Domestically produced goods become relatively less expensive pushing local demand of these goods up. In the absence of offsetting factors, the necessity to boost local production capacity raises prices for labor and capital leading domestic firms to increase their output prices (Orr, Scott, and White (1998)). Loss of access to international capital markets due to a lack of credibility is another explanation found for the fear of floating behavior. In this case, large swings in the exchange rate may have a contractionary effect in the economy causing an erosion of credibility that, depending on the degree, may result in a loss of access to international capital markets (Calvo and Reinhart (2000)). Further, Calvo and Reinhart (2002) believe in a combination of lack of credibility, high pass-through from exchange rates to prices, and inflation targeting as an additional reason why fear of floating may arise.

On the other hand, central banks have to consider that interventions on exchange rate depreciations are costly in the sense that they consume reserves. The cost of having low levels of reserves during an eventual financial crisis might exceed by far the costs generated by a more volatile currency although in many emerging markets large oscillations in the exchange rate can cause serious damage in the real economy as exports, imports and international capital flows are a relatively large share of the economy (Calvo and Mishkin (2003)). Additionally, if there is need to raise resources to finance the intervention, depending on how this is accomplished; the government can generate a substantial social cost. If the government has to use distortionary instruments like inflation tax to raise resources,[8] a welfare loss of the society will incur. Furthermore, countries with very elastic money demand, presumably countries with inflationary tradition, are most impacted by the use of this instrument. In order to avoid this cost, the government might finance the intervention by increasing lump-sum taxes or cutting unproductive expenditures. However, most of the time, this is not an option for governments as they already face fiscal constraints (Ganapolsky (2003)).

[8] Ganapolsky (2003) cites another distortionary instrument that might be used by governments to finance the intervention as such as to impose taxes on the financial system. Examples of countries that used this instrument are Brazil in 1999 and Argentina in 2001.

The implication that both interventions and depreciations of the currency are costly is that central banks face a tradeoff when deciding whether to intervene or not in the exchange market. How much to intervene will depend on each country's circumstances at a given point in time.

In this paper, I provide new insights about foreign exchange policy adopted by central banks around the world. It appears that central banks are more concerned about managing reserves rather than intervene in the market. Due to increasing globalization of financial markets the central banks apparently are foregoing of trying to stabilize the currency and prefer to keep high levels of reserves to fight financial crisis when they occur. The reason for this behavior is that with the new degree of interaction between the markets around the world it makes almost impossible to central banks to avoid financial crisis since now they can be originated from abroad which is out of control of the central banks. In this scenario, it is better to be prepared to fight crisis rather than try to avoid it.

2.3. Measuring the Degree of Intervention/Fueling by Central Banks

Interventions are characterized by an opposite movement in international reserves relative to an observed movement in the exchange rate. Any movement of the international reserves in the same direction of the exchange rate fuels the market.

Therefore, to measure the probability of an intervention and fueling episode occur, we have to consider four possible outcomes: a positive or a negative change in international reserves after an appreciation of the exchange rate and a positive or negative change in international reserves after a depreciation of the exchange rate.

First, lets S denote the space of all sign combinations between percent changes in exchange rates and percent changes in international reserves over a sample space from 1 to n. Then S can be represented by

$$\left\{\left\{\frac{\Delta ER_{t,t-1}}{ER_{t-1}}>0,\frac{\Delta IR_{t+1,t}}{IR_t}>0\right\}_{t=2}^{n},\left\{\frac{\Delta ER_{t,t-1}}{ER_{t-1}}>0,\frac{\Delta IR_{t+1,t}}{IR_t}<0\right\}_{t=2}^{n},\left\{\frac{\Delta ER_{t,t-1}}{ER_{t-1}}<0,\frac{\Delta IR_{t+1,t}}{IR_t}>0\right\}_{t=2}^{n},\left\{\frac{\Delta ER_{t,t-1}}{ER_{t-1}}<0,\frac{\Delta IR_{t+1,t}}{IR_t}>0\right\}_{t=2}^{n}\right\}$$

where t = 2,…,n. Each set of outcomes in the sample space which is a subset of S is an event. Denote event $\left\{\frac{\Delta ER_{t,t-1}}{ER_{t-1}} > 0, \frac{\Delta IR_{t+1,t}}{IR_t} > 0\right\}_{t=2}^{n}$ = $F1_t$, $\left\{\frac{\Delta ER_{t,t-1}}{ER_{t-1}} < 0, \frac{\Delta IR_{t+1,t}}{IR_t} < 0\right\}_{t=2}^{n}$ = $F2_t$, $\left\{\frac{\Delta ER_{t,t-1}}{ER_{t-1}} < 0, \frac{\Delta IR_{t+1,t}}{IR_t} > 0\right\}_{t=2}^{n}$ = $I1_t$, and $\left\{\frac{\Delta ER_{t,t-1}}{ER_{t-1}} > 0, \frac{\Delta IR_{t+1,t}}{IR_t} < 0\right\}_{t=2}^{n}$ = $I2_t$. It is easy to note that the set of outcomes associated with events $I1_t$ and $I2_t$ constitute the set of interventions in the exchange market for the sample space. Define n_1 as the total number of elements in $I1_t$ and n_2 as the total number of elements in $I2_t$. Additionally, events $F1_t$ and $F2_t$ contain all outcomes that characterize a fueling operation in the market. Consider w_1 as the total number of elements in $F1_t$ and w_2 as the total number of elements in $F2_t$. Note that the sum of n_1, n_2, w_1, and w_2 is equal to n-1.

Now, we can calculate the probability of each of these events occur by using conditional probabilities.

$$P(\textit{intervention type I}\,) = P\left(\Delta IR_{t,t-1} / IR_t > 0 \;\middle|\; \Delta ER_{t-1,t-2} / ER_{t-1} < 0\right) \tag{2.1}$$

$$P(\textit{intervention type II}) = P\left(\Delta IR_{t,t-1} / IR_t < 0 \;\middle|\; \Delta ER_{t-1,t-2} / ER_{t-1} > 0\right) \tag{2.2}$$

$$P(\textit{fueling type I}\,) = P\left(\Delta IR_{t,t-1} / IR_t > 0 \;\middle|\; \Delta ER_{t-1,t-2} / ER_{t-1} > 0\right) \tag{2.3}$$

$$P(\text{fueling } \textit{type II}) = P\left(\Delta IR_{t,t-1} / IR_t < 0 \;\middle|\; \Delta ER_{t-1,t-2} / ER_{t-1} < 0\right) \tag{2.4}$$

These probabilities provide important information about the behavior of central banks with respect to changes in the exchange rate. By calculating these terms, one can determine if there exists a predominant action taken by central banks.

Observe that the probability of intervention of type I and the probability of intervention of type II are calculated from two different sub-sample spaces. The first is taken from the sub-sample space of decreases in the exchange rate while the second is taken from the sub-sample space of increases in the exchange rate. Therefore, the total probability of intervention is not the sum of the individual probabilities but the sum of weighted probabilities relative to the sample space. The same argument applies for the calculation of the total probability of fueling the market.

Since the sum of the total probability of intervention and the total probability of fueling should be equal to 1 in the sample space, we can write an equation that contains all states of the world probabilities satisfying this correspondence.

$$\frac{(n_1 + w_1)}{(n-1)} P\left(\frac{\Delta IR_{t+1,t}}{IR_t} < 0 \;\middle|\; \frac{\Delta ER_{t+1,t}}{ER_{t-1}} > 0\right) + \frac{(n_2 + w_2)}{(n-1)} P\left(\frac{\Delta IR_{t+1,t}}{IR_t} > 0 \;\middle|\; \frac{\Delta ER_{t+1,t}}{ER_{t-1}} < 0\right) + \\ + \frac{(n_1 + w_1)}{(n-1)} P\left(\frac{\Delta IR_{t+1,t}}{IR_t} > 0 \;\middle|\; \frac{\Delta ER_{t+1,t}}{ER_{t-1}} > 0\right) + \frac{(n_2 + w_2)}{(n-1)} P\left(\frac{\Delta IR_{t+1,t}}{IR_t} < 0 \;\middle|\; \frac{\Delta ER_{t+1,t}}{ER_{t-1}} < 0\right) = 1 \quad (2.5)$$

The first two terms of equation (2.5) represent the probabilities of interventions of each type in the sample space and their sum denotes the total probability of intervention in the market.

$$\mathrm{P}(Int.) = \frac{(n_1 + w_1)}{(n-1)} P\left(\frac{\Delta IR_{t+1,t}}{IR_t} < 0 \;\middle|\; \frac{\Delta ER_{t+1,t}}{ER_{t-1}} > 0\right) + \frac{(n_2 + w_2)}{(n-1)} P\left(\frac{\Delta IR_{t+1,t}}{IR_t} > 0 \;\middle|\; \frac{\Delta ER_{t+1,t}}{ER_{t-1}} < 0\right) \quad (2.6)$$

Similarly, collecting the bottom terms of equation (2.5) gives us the total probability of fueling the market.

$$\mathrm{P}(Fuel) = \frac{(n_1 + w_1)}{(n-1)} P\left(\frac{\Delta IR_{t+1,t}}{IR_t} > 0 \;\middle|\; \frac{\Delta ER_{t+1,t}}{ER_{t-1}} > 0\right) + \frac{(n_2 + w_2)}{(n-1)} P\left(\frac{\Delta IR_{t+1,t}}{IR_t} < 0 \;\middle|\; \frac{\Delta ER_{t+1,t}}{ER_{t-1}} < 0\right) \quad (2.7)$$

Finally, using Bayes' rule, equation (2.6) and (2.7) can be rewritten as:

$$\mathrm{P}(Int.) = \frac{(n_1 + w_1)}{(n-1)} \frac{P\left(\frac{\Delta IR_{t+1,t}}{IR_t} < 0 \cap \frac{\Delta ER_{t+1,t}}{ER_{t-1}} > 0\right)}{P\left(\frac{\Delta ER_{t+1,t}}{ER_{t-1}} > 0\right)} + \frac{(n_2 + w_2)}{(n-1)} \frac{P\left(\frac{\Delta IR_{t+1,t}}{IR_t} > 0 \cap \frac{\Delta ER_{t+1,t}}{ER_{t-1}} < 0\right)}{P\left(\frac{\Delta ER_{t+1,t}}{ER_{t-1}} < 0\right)} \quad (2.8)$$

$$\mathrm{P}(Fuel) = \frac{(n_1 + w_1)}{(n-1)} \frac{P\left(\frac{\Delta IR_{t+1,t}}{IR_t} > 0 \cap \frac{\Delta ER_{t+1,t}}{ER_{t-1}} > 0\right)}{P\left(\frac{\Delta ER_{t+1,t}}{ER_{t-1}} > 0\right)} + \frac{(n_2 + w_2)}{(n-1)} \frac{P\left(\frac{\Delta IR_{t+1,t}}{IR_t} < 0 \cap \frac{\Delta ER_{t+1,t}}{ER_{t-1}} < 0\right)}{P\left(\frac{\Delta ER_{t+1,t}}{ER_{t-1}} < 0\right)} \quad (2.9)$$

Because interventions smooth out changes in the exchange rate and fueling the market increases volatility, we should expect total probability of intervention be much greater than total probability of fueling the market for countries that fear to let their currency float. Then, if really there is a spread fear of floating among countries, we anticipate that this behavior should be consistent across countries.

$$P(\textit{intervention} \mid \textit{Fear of floating}) >> P(\textit{fueling} \mid \textit{Fear of floating}) \qquad (2.10)$$

Furthermore, since central banks frequently operate in the market due to "normal" market operations,[9] we should expected total probability of intervention be approximately the same to total probability of fueling the market for countries that do not fear to let their currency float.

$$P(\textit{intervention} \mid \textit{Float}) \approx P(\textit{fueling} \mid \textit{Float}) \qquad (2.11)$$

In addition, we have to consider the size of the intervention/fueling adopted by central banks. This is also an important measure since the overall degree of intervention/fueling depends not only on the probability of intervention/fueling but also on the amount of international reserves used to intervene/fuel in the market.

The conditional expected value of the change in international reserves for each event can be computed by calculating its average over the corresponding sample space.

$$E\left(\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}} > 0, \frac{\Delta IR_{t+1,t}}{IR_t} < 0\right) = \frac{1}{n_1}\sum_{t=2}^{n}\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}} > 0, \frac{\Delta IR_{t+1,t}}{IR_t} < 0 \qquad (2.12)$$

$$E\left(\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}} < 0, \frac{\Delta IR_{t+1,t}}{IR_t} > 0\right) = \frac{1}{n_2}\sum_{t=2}^{n}\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}} < 0, \frac{\Delta IR_{t+1,t}}{IR_t} > 0 \qquad (2.13)$$

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[9] By normal market operations I mean transactions effectuated by central banks that do not seek to intervene in the exchange rate (although it might) but rather are result of contracts requirements that should be honored by central banks. As mentioned by Calvo and Reinhart (2002): "in the case of New Zealand, reserves fluctuate due to the Treasury's management of its overseas currency debt rather than foreign exchange market intervention."

$$E\left(\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}>0,\frac{\Delta IR_{t+1,t}}{IR_t}>0\right)=\frac{1}{w_1}\sum_{t=2}^{n}\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}>0,\frac{\Delta IR_{t+1,t}}{IR_t}>0 \tag{2.14}$$

$$E\left(\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}<0,\frac{\Delta IR_{t+1,t}}{IR_t}<0\right)=\frac{1}{w_2}\sum_{t=2}^{n}\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}<0,\frac{\Delta IR_{t+1,t}}{IR_t}<0 \tag{2.15}$$

Equations (2.12), (2.13), (2.14), and (2.15) denote the conditional expected value of the change in international reserves for intervention type I, intervention type II, fueling type I, and fueling type II, respectively.

Finally, combining equations found for the probability of each event occur (equations 2.1-2.4) and the corresponding conditional expected value of the change in international reserves, one can determine the degree of intervention and fueling (*D*) for each state of the world.

$$D(Int.\ Type\ I)=P\left(\frac{\Delta IR_{t+1,t}}{IR_t}<0\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}>0\right)\cdot E\left(\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}>0,\frac{\Delta IR_{t+1,t}}{IR_t}<0\right) \tag{2.16}$$

$$D(Int.\ Type\ II)=P\left(\frac{\Delta IR_{t+1,t}}{IR_t}>0\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}<0\right)\cdot E\left(\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}<0,\frac{\Delta IR_{t+1,t}}{IR_t}>0\right) \tag{2.17}$$

$$D(Fuel.\ Type\ I)=P\left(\frac{\Delta IR_{t+1,t}}{IR_t}>0\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}>0\right)\cdot E\left(\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}>0,\frac{\Delta IR_{t+1,t}}{IR_t}>0\right) \tag{2.18}$$

$$D(Fuel.\ TypeII)=P\left(\frac{\Delta IR_{t+1,t}}{IR_t}<0\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}<0\right)\cdot E\left(\frac{\Delta IR_{t+1,t}}{IR_t}\middle|\frac{\Delta ER_{t,t-1}}{ER_{t-1}}<0,\frac{\Delta IR_{t+1,t}}{IR_t}<0\right) \tag{2.19}$$

These equations provide an indicator of the policy adopted by central banks of using international reserves to control fluctuations in the exchange rate. They not only incorporate the probability of an action by a central bank to swings in the exchange rate but consider the amount used as well. Additionally, by clearly separating the possible movements in international reserves after changes in the exchange rate, this method avoids the omitted variable bias observed in most studies that try to measure the degree of intervention by looking only at the volatility of the exchange rate and of international reserves. The drawback of this method is that it does not relate the degree of the change of international reserves with the degree of the change of the exchange rate. To fulfill this gap, further analysis evaluates the correlation between international reserves and exchange rates for the two possible occurrences (appreciation and depreciation) using panel data.

2.4. Is fear of floating really out there?

The data consists of monthly percent changes in international reserves and exchange rates of 31 countries from all parts of the world including Africa, Asia, Oceania, Europe, North America, and Latin America. Countries included are Australia, Bolivia, Brazil, Canada, Chile, Colombia, Egypt, Greece, India, Indonesia, Israel, Japan, Kenya, Korea, Malaysia, Mexico, New Zealand, Nigeria, Norway, Pakistan, Peru, Philippines, Singapore, South Africa, Spain, Sweden, Turkey, Uganda, United States, Uruguay, and Venezuela. The time horizon varies country-by-country depending on data availability, the exchange rate regime of interest, and the inflation in the period. The sample basically resembles the sample of managed and free floating countries used by Calvo and Reinhart (2000), expanded through December of 2006.[10] Most exchange rates and, consequently, international reserves are taken in United States dollar (US$). Exceptions are countries from Europe and the United States (prior January of 1999) in which the exchange rate used is against the German deutsche mark (DM) and United States (post European Union) which the exchange rate is against the euro (€). The objective is to investigate the behavior of the international reserves parallel to the movements of the exchange rates to verify whether countries claimed to be free or managed floaters are really presenting fear of floating, as observed by others. This study examines only interventions through the use of reserves. Although changes in the interest rate may also have an indirect impact on the exchange market, changing the amount of foreign currency supplied or demanded has a more immediate and direct effect on the exchange rate.

We begin by analyzing the mean and the volatility (measured by standard deviation) of exchange rates and international reserves of the sample. Table 2.1 shows the results for countries classified as free floaters. Overall, compared to developed markets, emerging markets present higher and positive expected monthly exchange rate change (indicating a tendency of depreciation of the domestic currency) and higher exchange rate volatility. Individually, however, some developing countries, especially Bolivia and

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[10] For the sample expansion period, the regime classification followed the IMF *de facto* classification provided since 1999.

India, present relatively low exchange rate volatility compared to some developed countries. The same pattern is observed between Latin American markets and developed markets. In terms of international reserves, it can be noted that there is a tendency to increase reserves among countries. The monthly expected value for this parameter is positive, although in different degrees, for all countries in the sample. The standard deviation of international reserves is highest for emerging markets, followed by Latin American markets. It is worth to mention that some developing countries, like Bolivia, Nigeria, Philippines, and South Africa, present relatively much higher volatility of foreign reserves.

Finally, the last column presents a measure of international reserve volatility per volatility unit of exchange rate. Countries with high oscillations in foreign reserves and low exchange rate variability should have therefore large values for this parameter. According to Calvo and Reinhart (2002), this fact evidences that the central bank is intervening in the exchange market. Following this argument, we could say that among the countries with higher extent of intervention in the foreign exchange market are Bolivia, Canada, Norway, Philippines, South Africa, and Uganda. On the other extreme, with the most independent monetary policy, is the United States after the Euro zone was implemented. At the aggregate level, we find that the Latin American markets have, overall, the highest international reserve volatility per volatility unit of exchange rate.

Table 1: Mean and standard deviation of international reserves and exchange rate change for countries adopting free floating regimes (monthly percent change).

		International Reserves		Exchange Rate		Relative Volatility
Country	Period	Mean	Std. dev.	Mean	Std. dev.	σ (int. res.) / σ (exch. rate)
Australia	Jan/84-Dec/06	0.92	7.04	0.11	3.06	2.30
Bolivia	Set/85-Dec/97	3.29	21.85	0.89	1.38	15.83
Brazil	Jan/99-Dec/06	1.33	7.41	0.38	5.61	1.32
Canada	Jun/70-Dec/06	1.08	11.35	0.04	1.44	7.88
Chile	Set/99-Dec/06	0.40	3.16	0.01	2.69	1.17
Colombia	Dec/99-Dec/06	0.82	2.34	0.19	2.45	0.96
India	Mar/93-Dec/99	2.10	4.89	0.42	1.40	3.49
Japan	Feb/73-Dec/06	1.06	3.72	-0.15	3.19	1.17
Kenya	Oct/93-Dec/97	2.04	11.40	-0.04	5.21	2.19
Korea	Nov/97-Dec/06	2.23	2.90	-0.41	3.17	0.91
Mexico	Dec/94-Dec/06	1.65	6.27	0.49	3.28	1.91
New Zealand	Mar/85-Dec/06	1.36	9.20	-0.12	3.25	2.83
Nigeria	Oct/86-Mar/93	1.90	19.95	3.14	10.52	1.90
Norway	Dec/92-Dec/94	1.24	5.61	0.11	0.92	6.10
Peru	Aug/90-Dec/03	1.51	5.42	1.37	3.67	1.48
Philippines	Jan/88-Dec/06	2.17	12.68	0.41	2.71	4.68
South Africa	Jan/83-Dec/06	3.46	22.24	0.75	4.53	4.91
Spain	Jan/84-May/89	2.56	4.80	0.26	3.39	1.42
Sweden	Nov/92-Dec/06	0.25	6.39	0.06	1.71	3.74
Turkey	Feb/01-Dec/06	1.92	5.19	0.44	5.10	1.02
Uganda	Jan/92-Dec/06	2.47	9.92	0.25	2.20	4.51
United States (DM)	Feb/73-Dec/98	1.01	5.47	0.23	2.83	1.93
United States (Euro)	Jan/99-Dec/06	-0.34	2.11	0.04	2.78	0.76
Developed Markets	-	0.99	7.53	0.04	2.79	2.70
Emerging Markets	-	2.04	12.77	0.64	4.40	2.90
East Asia	-	1.31	3.59	-0.20	3.19	1.13
Latin America	-	1.67	11.04	0.67	3.38	3.27

Source: International Financial Statistics; International Monetary Fund.

Table 2.2 presents the same statistics for managed floating countries in the sample. The results, at the aggregate level, follow closely the evidence found for free floating countries. Emerging markets have the highest volatility of international reserves but also the highest volatility of exchange rates. The expected monthly change in international reserves is also positive for all countries. Countries that supposedly intervene more in the foreign exchange market (large value for international reserve volatility per volatility unit of exchange rates) are Bolivia, Egypt, Mexico, Pakistan, Brazil, Korea, and Venezuela. The lowest value found for this parameter belongs to Chile. Although the international reserve volatility per volatility unit of exchange rate for emerging markets is lower than for developed markets, it should be addressed that the developed market index in this sample is composed by only three countries (Greece, Norway, and Singapore) as not many developed countries has opted for the managed float arrangement.

Comparing the statistics found for free and managed floaters, one can observe that for all markets the standard deviation of the changes in exchange rate is greater for the free floating regime sample, which corroborates with the theory that free floaters face larger exchange rate fluctuations than managed floaters. Furthermore, we should expect larger swings of foreign reserves for managed floaters as it is expected that they will use this instrument more heavily to control the volatility of the exchange rate. This fact is indeed observed for developed and East Asia markets, but not for emerging and Latin American markets which showed the opposite result. In terms of the extent of intervention in the exchange market, measured here by international reserve volatility per volatility unit of exchange rates, all markets, except for Latin America, present larger value of this parameter for the corresponding managed floating regime.

Table 2: Mean and standard deviation of international reserves and exchange rate change for countries adopting managed floating regimes (monthly percent change).

		International Reserves		Exchange Rate		Relative Volatility
Country	Period	Mean	Std. dev.	Mean	Std. dev.	σ (int. res.) / σ (exch. rate)
Bolivia	Jan/98-Dec/06	1.26	7.88	0.36	0.31	25.42
Brazil	Jul/94-Dec/98	0.39	8.22	0.69	0.8	10.28
Chile	Oct/82-Aug99	1.2	5.5	1.19	5.09	1.08
Colombia	Jan/79-Nov/99	0.71	6.84	1.57	1.76	3.89
Egypt	Feb/91-Dec/98	1.8	3.91	0.05	0.19	20.58
Greece	Jan/77-Dec/97	1.68	12.14	0.96	2.28	5.32
India	Feb/79-Nov/93	0.68	11.5	0.79	2.57	4.47
Indonesia	Nov/78-Jun/97	1.12	7.02	0.68	3.89	1.80
Israel	Dec/91-Apr/04	1.17	5.98	0.47	1.96	3.05
Kenya	Jan/98-Dec/06	1.12	3.92	0.16	1.83	2.14
Korea	Mar/80-Oct/97	1.48	7.7	0.23	0.95	8.11
Malaysia	Dec/92-Sep/98	0.54	7.04	0.65	4.5	1.56
Mexico	Jan/89-Nov/94	2.23	12.73	0.56	0.95	13.40
Nigeria	Jan/99-Dec/06	2.43	6.05	0.39	1.46	4.14
Norway	Jan/95-Dec/01	0.14	6.7	-0.09	1.4	4.79
Pakistan	Jan/82-Dec/06	3.07	22.25	0.58	1.68	13.24
Singapore	Jan/88-Dec/06	0.98	1.94	-0.11	1.53	1.27
Turkey	Jan/80-Jan/01	1.87	10.73	3.81	4.28	2.51
Uruguay	Jan/93-Mar/03	0.63	9.88	1.79	3.82	2.59
Venezuela	Apr/96-Dec/01	0.55	6.3	0.74	0.83	7.59
Developed Markets	-	1.17	8.62	0.37	1.96	4.40
Emerging Markets	-	1.5	11.14	1.08	3.08	3.62
East Asia	-	1.13	6.16	0.3	2.75	2.24
Latin America	-	0.97	7.84	1.17	3.06	2.56

Source: International Financial Statistics; International Monetary Fund.

So far, we showed that although the aggregate data results are in line with the expected monetary policy to be adopted by central banks taking into account their official arrangement, at the individual level that might not be true. The preliminary results demonstrate that some countries, developed or not, are, in principle, resilient to let their currency to float which supports the fear of floating hypothesis. The results above were highly expected, since the sample used incorporates the same countries adopted by Calvo and Reinhart (2002) and the methodology applied is in some sense very similar.

However, there is a potential drawback in comparing volatility of reserves and exchange rate volatilities as an indicator of the extend central banks are intervening in the market. Some countries may have more volatile exchange rate simply because they are exposed to larger external shocks even if the central bank intervene heavily to defend its parity (Hausmann, Panizza, and Stein (2000)). Also, we might observe large volatility in reserves due to persistent increases in its level which not necessarily imply that interventions are taking place.

The next step is to use a different approach to calculate the degree of intervention in the foreign exchange market by central banks. The idea is to confront the results of the new method with the largely accepted fear of floating hypothesis. As shown in the previous section, the method involves calculating the expected value of the movement in foreign reserves given an observed increase (depreciation) or decrease (appreciation) in the exchange rate. The process involves to first calculating the probability of each possible state to happen. That is, the probability of an increase/decrease of the international reserves given an increase/decrease in the exchange rate. As mentioned before, only two combinations of these possible states characterize an intervention (when international reserves move in the opposite direction of exchange rate) reducing volatility, while the other two characterize "fuel" the market (when the international reserves move in the same direction of the exchange rate) increasing volatility.

First, let's take a look at the probabilities for each event to occur at the aggregate level. It will give us a good indication of what is ahead of us. Table 2.3 shows the probabilities for the two possible types of intervention the central bank may engage. Intervention type I (column 2) refers to a decrease in international reserves given an increase in the exchange rate (depreciation of the domestic currency). Intervention type II (column 3) refers to an increase in international reserves given a decrease in the exchange rate. Fueling type I (column 5) and fueling type II (column 6) are the counterparts of intervention type I and II, respectively.[11] The total probability of intervention (column 4) and fueling (column 7) are simply the weighted average of the two types of intervention and the two types of fueling, respectively. The probabilities are taken considering a lag of one month for changes in the exchange rate and changes in international reserves. For example, the probability of intervention type I is the probability that reserves at the present month (*t*) will decrease given that the change in the exchange rate was positive in the previous month (*t-1*). The rationale for using the lag is that so we avoid the simultaneity bias that would certainty occur if we take changes in the same month for both parameters as changes in international reserves directly affects the level of the exchange rate.[12]

[11] Intervention type I, intervention type II, fueling type I, and fueling type II are calculated using equations 16, 17, 18, and 19, respectively.

[12] On the other hand, using a lag of one month to calculate these probabilities might loose track of the timing of events as central banks operate on daily basis. In our judgment, the simultaneity bias is more pervasive in this case because directly affects the parameters used although the same analysis was done using same month data and the results obtained are similar.

Table 2.3: Probability of intervention and fueling the market by central banks for free and managed floating exchange rate regimes.

Markets	Probability of intervention Type I $P(\Delta IR/IR<0\ \Delta ER/ER>0)$	Probability of intervention Type II $P(\Delta IR/IR>0\ \Delta ER/ER<0)$	Total probability of intervention	Probability of fueling Type I $P(\Delta IR/IR>0\ \Delta ER/ER>0)$	Probability of Fueling Type II $P(\Delta IR/IR<0\ \Delta ER/ER<0)$	Total probability of fueling
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: Free floaters						
All countries	46.2***	64.7***	54.9***	53.8***	35.3***	45.1***
Developed Markets	47.5	63.0***	55.3***	52.5	37.0***	44.7***
Emerging Markets	43.5***	66.5***	54.2***	56.5***	33.5***	45.8***
East Asia	30.3***	86.5***	58.7***	69.7***	13.5***	41.3***
Latin America	43.8**	62.4***	51.1	56.2**	37.6***	48.9
Panel B: Managed floaters						
All countries	45.7***	63.5***	51.3	54.3***	36.5***	48.7
Developed Markets	46.5	67.6***	55.5***	53.5	32.4***	44.5***
Emerging Markets	45.3***	61.7***	50.1	54.7***	38.3***	49.9
East Asia	40.4***	72.1***	53.0	59.6***	27.9***	47.0
Latin America	45.3**	65.9***	48.7	54.7**	34.1***	51.3

Source: International Financial Statistics; International Monetary Fund.

****, **, and * - Paired t-test between columns (3) and (6), (4) and (7), and (5) and (8) for mean difference statistically significant at the 1% 5% and 10% level, respectively.*

The next analysis combines the probability of each intervention/fueling to happen with the size of the change in international reserves. The expected degree of intervention/fueling is calculated by multiplying the probability of each event occur with the expected value of the international reserve change for each event as well (estimated by the mean). In order to capture the differences in policy among the countries and to better understand the reasoning behind the central banks' actions, the results are now shown in both the aggregate level and for each country individually (table 2.4).

At the aggregate level, incorporating the size of the change in international reserves does not modify the tendency observed for the probabilities. The results still show, for most cases, higher expected intervention relative to expect fueling generated by larger difference between the expected amount of increase reserves and the expected decrease in reserves after an appreciation of the exchange rate relative to the difference between the expected amount of increase in reserves and the expected decrease in reserves after a depreciation of the exchange rate.

Table 2.4: Expected intervention/fueling by central banks for free floating exchange rate regime.

Country	Period	Exp. Int. Type I $E(\Delta IR/IR<0$ $\Delta ER/ER>0)$	Exp. Int. Type II $E(\Delta IR/IR>0$ $\Delta ER/ER<0)$	Expected intervention	Exp. Fueling Type I $E(\Delta IR/IR>0$ $\Delta ER/ER>0)$	Exp. Fueling Type II $E(\Delta IR/IR<0$ $\Delta ER/ER<0)$	Expected fueling
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Australia	Jan/84-Dec/06	-2.41	3.30	2.87	2.32	-1.46	1.88
Bolivia	Set/85-Dec/97	-5.63	5.90	5.66	9.00	-3.61	8.40
Brazil	Jan/99-Dec/06	-1.55	3.24	2.41	2.87	-2.02	2.44
Canada	Jun/70-Dec/06	-2.50	3.44	2.96	3.92	-2.70	3.32
Chile	Set/99-Dec/06	-0.59	0.88	0.73	1.48	-1.03	1.26
Colombia	Dec/99-Dec/06	-0.52	1.65	1.12	0.92	-0.46	0.68
India	Mar/93-Dec/99	-1.13	3.78	2.36	1.87	-0.31	1.15
Japan	Feb/73-Dec/06	-0.93	2.45	1.67	0.90	-0.26	0.59
Kenya	Oct/93-Dec/97	-6.09	8.62	7.79	2.46	-1.62	1.90
Korea	Nov/97-Dec/06	-0.06	2.63	1.56	2.06	-0.03	0.87
Mexico	Dec/94-Dec/06	-1.31	2.32	1.77	4.38	-0.59	2.63
New Zealand	Mar/85-Dec/06	-2.71	4.56	3.77	3.06	-2.45	2.71
Nigeria	Oct/86-Mar/93	-5.76	6.33	5.96	8.54	-5.42	7.43
Norway	Dec/92-Dec/94	-1.23	2.85	2.08	2.53	-1.75	2.12
Peru	Aug/90-Dec/03	-0.78	1.41	1.03	3.10	-0.91	2.23
Philippines	Jan/88-Dec/06	-2.64	5.05	3.72	4.15	-2.02	3.20
South Africa	Jan/83-Dec/06	-5.06	7.38	6.06	8.78	-4.74	7.04
Spain	Jan/84-May/89	-0.56	3.34	1.98	3.22	-0.79	1.98
Sweden	Nov/92-Dec/06	-2.74	2.95	2.84	1.85	-1.54	1.69
Turkey	Feb/01-Dec/06	-1.62	3.38	2.64	2.59	-0.85	1.58
Uganda	Jan/92-Dec/06	-1.56	5.08	3.01	2.77	-1.05	2.07
United States (DM)	Feb/73-Dec/98	-1.71	3.12	2.40	1.67	-0.97	1.33
United States (Euro)	Jan/99-Dec/06	-1.00	0.63	0.81	0.55	-0.72	0.65
All countries	-	**-2.32**	**3.57**	**2.90**	**3.54**	**-1.64**	**2.64**
Developed Markets	-	**-1.90**	**3.13**	**2.51**	**2.25**	**-1.49**	**1.87**
Emerging Markets	-	**-2.36**	**3.86**	**3.06**	**4.46**	**-1.89**	**3.26**
East Asia	-	**-0.78**	**2.49**	**1.64**	**1.05**	**-0.18**	**0.61**
Latin America	-	**-2.36**	**2.16**	**2.28**	**4.44**	**-1.13**	**3.14**

Source: International Financial Statistics; International Monetary Fund.

As we can observe, the total probability of intervention is greater than the total probability of fueling for all markets, free and managed floaters, with only one exception which is the Latin American managed floaters although the difference in probabilities in this case did not reach statistical significance at the 10% level. For most other cases, the difference in means test is highly significant which gives support to the fear of floating hypothesis. However, when we decompose these total probabilities into the different types of intervention and fueling, we start seeing a different picture of what is really happening in terms of exchange policy. Supported by the difference in means test in all cases (at the 1% level), we observe a strong tendency in increasing international reserves relatively to decrease reserves after a drop in the exchange rate. Furthermore, there is also a higher probability of an increase in international reserves after a depreciation of the domestic currency (probability of fueling type I) as opposed to a decrease of the international reserves which characterizes an intervention. For all aggregate indexes the probability of fueling type I is greater than the probability of intervention type I and, except for developed markets, the difference in means test is significant at the 5% level, at least. These results indicate that the difference between the total probability of intervention and the total probability of fueling was a result of a very large difference in probabilities between intervention type II and fueling type II bringing the wrong conclusion that central banks are constantly intervening in the exchange market.

In fact, what can be perceived with these results is that the main goal of the central banks is to increase reserves no matter the movement in the exchange rate. Here the results start going against the fear of floating hypothesis. Yet, only the probabilities of the actions are not enough to support the argument that increasing reserves is the main goal of the central banks. We also have to consider the size of each action in order to capture the real degree of intervention/fueling the central banks are taking..

Also, emerging markets increase reserves more than developed markets no matter the direction of the change in the exchange rate in the previous month. The difference is especially greater after a depreciation of the domestic currency (4.46% for emerging markets and 2.25% for developed markets). This result is just the opposite of what the fear of floating hypothesis predicts. Here, increasing reserves prevails over stabilizing the currency. This is probably because during financial crises emerging markets experience shocks to its external supply. In this case, international reserves act as an insurance instrument against the pressure on the domestic currency. Since crises are characterized by foreign currency shortages, there is an incentive to central banks to keep international reserves in levels that allow them to ease shortages when they occur.

Interestingly, for emerging markets and Latin American markets, the expected increase in international reserves after a depreciation of the domestic currency is greater than after an appreciation. This is most evidenced for Bolivia (Set/85-Dec/97), Mexico (Dec/94-Dec/06), Nigeria (Oct/86-Mar/93), Peru (Aug/90-Dec/03), and South Africa (Jan/83-Dec/06). This behavior is, in principle, unintuitive as we would expect that countries willing to buy reserves will do it most after an appreciation of the domestic currency that is when the foreign currency becomes cheaper. However, we can think that these markets faced constant pressure on the depreciation side of their currency in the period of analysis, not to mention eminency of an eventual financial crisis. In this case, if a further depreciation is expected, buying reserves even after a depreciation of the domestic currency is cheaper than buying later. Additionally and most importantly, composing reserves act as insurance if a financial crisis takes place which is more likely to happen in countries already suffering devaluation of their currency. Nevertheless, the willingness to accumulate reserves despite the movement in the exchange rate is also observed among some developed countries like Canada and Spain in the 80's. With no specific pattern and very low values for the expected changes in international reserves is the United States after the euro zone was created. Japan also has low expected responses to exchange rate movements relative to developed countries.

For managed floaters, the phenomenon of willingness to increase reserves despite the movement in the exchange rate is also observed (table 2.5). Once again, for emerging markets, especially Egypt (Feb/91-Dec/98), Mexico (Jan/89-Nov/94), Nigeria (Jan/99-Dec/06), Pakistan (Jan/82-Dec/06), and Turkey (Jan/80-Jan/01), this behavior is more preeminent. This makes sense since those countries were shaken by international financial crises in these periods. Greece also seems to adopt the policy of primarily increasing reserves. The expected value of an increase in international reserves after a appreciation and depreciation of the exchange rate is much higher than its counterparts (4.74% and 5.27% against -2.00% and -3.98%, respectively). This behavior, although in different levels, is followed by most countries.

Table 2.5: Expected intervention/fueling by central banks for managed floating exchange rate regime.

Country (1)	Period (2)	Exp. Int. Type I $E(\Delta IR/IR<0$ $\Delta ER/ER>0)$ (3)	Exp. Int. Type II $E(\Delta IR/IR>0$ $\Delta ER/ER<0)$ (4)	Expected intervention (5)	Exp. Fueling Type I $E(\Delta IR/IR>0$ $\Delta ER/ER>0)$ (6)	Exp. Fueling Type II $E(\Delta IR/IR<0$ $\Delta ER/ER<0)$ (7)	Expected fueling (8)
Bolivia	Jan/98-Dec/06	-2.99	5.62	3.51	3.18	-0.04	2.56
Brazil	Jul/94-Dec/98	-2.32	0.15	2.07	3.04	-2.15	2.94
Chile	Oct/82-Aug99	-1.50	2.52	1.76	2.52	-0.65	2.05
Colombia	Jan/79-Nov/99	-1.83	1.74	1.82	2.52	-0.84	2.37
Egypt	Feb/91-Dec/98	-0.47	1.71	1.14	2.81	-0.29	1.46
Greece	Jan/77-Dec/97	-3.98	4.74	4.18	5.27	-2.00	4.40
India	Feb/79-Nov/93	-3.18	4.63	3.66	3.18	-2.53	2.96
Indonesia	Nov/78-Jun/97	-1.79	4.79	2.37	2.59	-2.06	2.48
Israel	Dec/91-Apr/04	-1.81	3.05	2.39	2.08	-0.87	1.52
Kenya	Jan/98-Dec/06	-0.96	2.26	1.59	1.99	-1.00	1.50
Korea	Mar/80-Oct/97	-2.33	4.59	3.25	2.69	-1.46	2.20
Malaysia	Dec/92-Sep/98	-2.80	1.82	2.32	2.65	-0.66	1.68
Mexico	Jan/89-Nov/94	-3.70	3.38	3.62	5.90	-1.70	4.81
Nigeria	Jan/99-Dec/06	-1.26	4.13	2.62	3.37	-1.68	2.57
Norway	Jan/95-Dec/01	-2.55	3.10	2.84	2.23	-2.27	2.25
Pakistan	Jan/82-Dec/06	-5.50	8.72	6.85	8.41	-5.33	7.12
Singapore	Jan/88-Dec/06	-0.45	1.51	1.05	1.10	-0.28	0.63
Turkey	Jan/80-Jan/01	-3.20	10.55	3.73	4.56	-1.19	4.32
Uruguay	Jan/93-Mar/03	-2.43	1.41	2.34	3.53	-4.83	3.65
Venezuela	Apr/96-Dec/01	-2.40	6.88	3.07	2.11	-0.65	1.89
All countries	-	**-2.59**	**4.07**	**3.05**	**3.60**	**-1.75**	**3.02**
Developed Markets	-	**-2.73**	**2.71**	**2.72**	**3.61**	**-1.12**	**2.54**
Emerging Markets	-	**-2.61**	**4.74**	**3.22**	**3.67**	**-2.06**	**3.20**
East Asia	-	**-1.73**	**2.91**	**2.20**	**2.30**	**-0.93**	**1.75**
Latin America	-	**-2.19**	**3.22**	**2.36**	**2.99**	**-1.06**	**2.67**

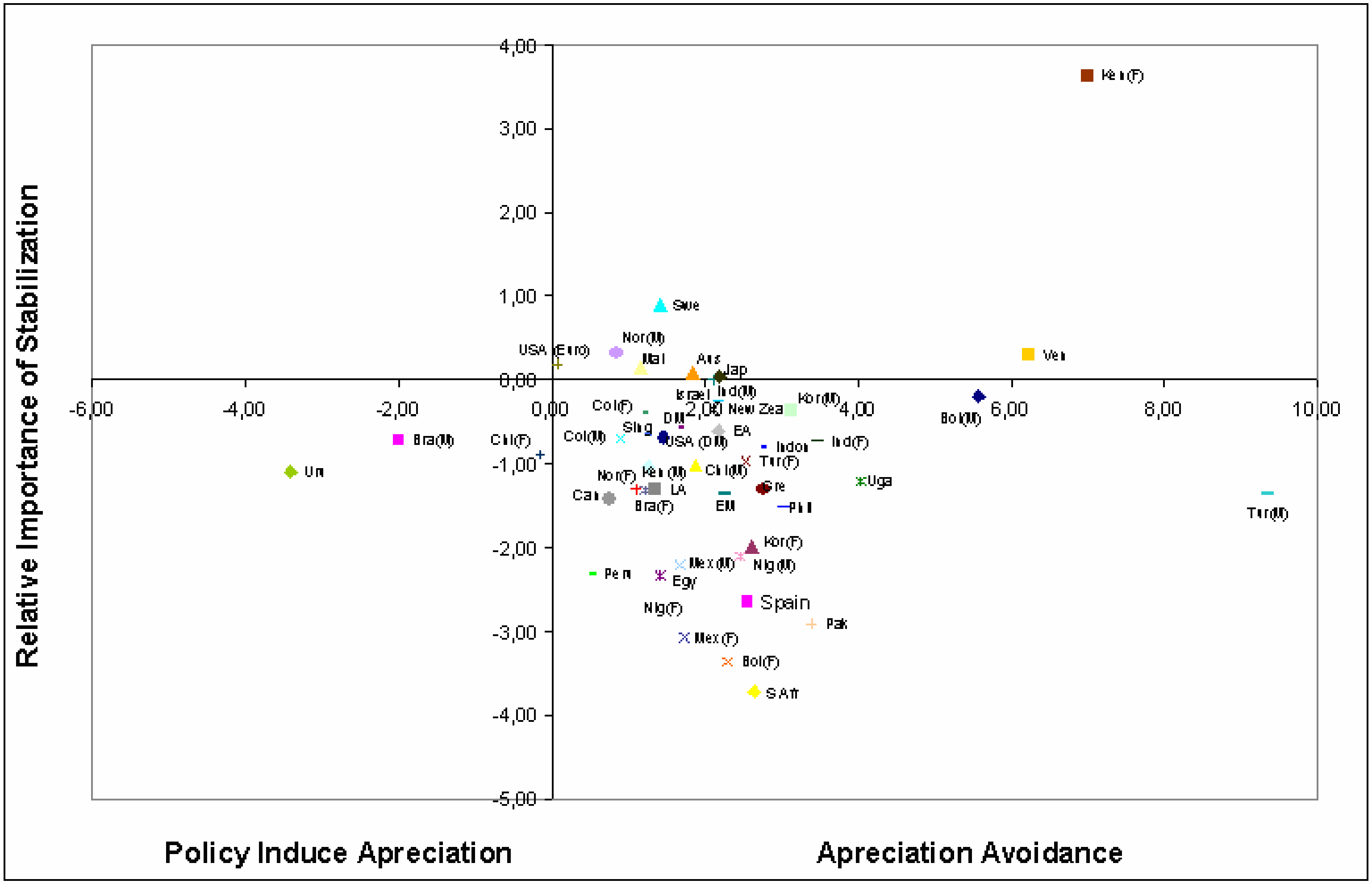
Source: International Financial Statistics; International Monetary Fund.

Figure 2.1 illustrates the main point of this paper. The horizontal axis represents the difference in absolute value between the expected value of the increase and the decrease in international reserves after an appreciation of the exchange rate.[13] Therefore, countries showing positive value for this parameter are engaged in appreciation avoidance of the domestic currency. That is, increasing reserves after a decrease in the exchange rate prevails over decreasing reserves. Inversely, a negative value of this difference means the central bank is engaged in a policy to induce appreciation. The vertical axis represents the relative importance of stabilization of the domestic currency. Negative values in this case indicate that buying reserves after a depreciation of the domestic currency prevails over selling reserves. Here, free floaters and managed floaters countries are all in the same graph. This facilitates comparisons of behavior for same country when adopting different regimes.

Clearly, there is a tendency for countries to locate on the right-bottom side of the graph. Overall, countries try to avoid appreciation of the domestic currency and give relative low importance for stabilization of their currency. These are the cases when the central bank tends to buy reserves after the exchange rate has depreciated and appreciated, respectively.

[13] The idea here is the same as summing up vector forces to see which tends to be the strongest and then observe to which direction the resulting vector is pointing.

Figure 2.1: Expected intervention/fueling by central banks for countries adopting free and managed floating exchange rate regimes.



Interestingly, some countries engaged more heavily in the policy of accumulating reserves when they are free floater than when they are adopting the managed float regime. This is clearly the case of Brazil (Jan/99-Dec/06), India (Mar/93-Dec/99), Korea (Nov/97-Dec/06), Mexico (Dec/94-Dec/06), and Norway (Dec/92-Dec/94). Others countries go in the opposite direction, like Chile (Oct/82-Aug/99), Kenya (Jan/98-Dec/06), and Turkey (Jan/80-Jan/01). Nevertheless, most of these countries share the same pattern of increasing the degree of reserve accumulation after being hit by a financial crisis.[14] This is a reasonable behavior since emerging markets experience foreign currency shortages during financial shocks. The experience of suffering from a sudden stop of capital inflow leaded these markets to adopt this policy even if it is costly. We also observe higher frequency of developed countries close to the centre of the graph. The United States after the Euro is the only country with absolute no specific trend in the exchange policy. Japan and Australia adopt the policy of appreciation avoidance and no explicit policy in terms of stabilization of their currency. In much higher degree, Venezuela and Bolivia, both managed floaters, follow the same pattern. As Venezuela is highly economic dependent of its oil exports, depreciating the currency increases revenues. As an extreme case, Kenya, when adopting the free float regime, is the only country where the intervention policy is clearly taking place in both sides of the exchange rate change. Relative importance of stabilization and appreciation avoidance are both

[14] Chile suffered a major financial crisis in 1982, Mexico in 1994-1995, Korea in 1997, Brazil in 1998-1999, Norway was affected by the crisis in the European Monetary System in 1992, and India suffered spillover from the East Asia crisis started in 1997.

very high for this country. With the exact opposite trend, Brazil and Uruguay, both when adopted the managed float regime, engaged in a policy induce appreciation with low relative importance of stabilization. We also observe that Turkey, when adopting the managed float regime, engaged in a massive reserves accumulation through appreciation avoidance. In terms of aggregate markets,[15] developed markets (DM) are closer to the centre of the graph as expected. East Asia (EA) countries adopt about the same level of relative importance of stabilization as developed countries but avoid appreciation of their currency more heavily. Latin American (LA) countries have lower relative importance of stabilization value than developed countries. Finally, emerging market (EM) countries have the highest degree of reserve accumulation for both parameters in the graph.

Our results so far show that the idea that there is a spread fear of floating among countries around the world is misleading. If the main concern of the central banks is to smooth out the changes in the exchange rate, we would observe a tendency of selling reserves after a depreciation of the domestic currency. However, what we really observe is the opposite which cause an increase of the exchange rate volatility. Further, this behavior at the up side of the exchange rate movement makes the problem of currency mismatch even worse which is the main support for the fear of floating hypothesis. The fact that central banks also opt for buying reserves after a depreciation of the domestic currency make us to believe that central banks are constantly looking for buying

[15] Here the market indexes are composed by countries of both regimes (free and managed floaters). Because it was observed the same pattern for both regimes, I decided to combine them so comparisons are made simpler.

insurance against financial crisis. It looks like central banks put a high price on being prepared when a financial crisis hit the economy even though that might contribute for such crisis to happen. The logic here is that is better be prepared when a crisis happen rather than try to avoid it.

Our final analysis incorporates the level of the changes in the exchange rate with changes in international reserves. In order to capture the correlation between these two variables, we use panel data analysis (for a heteroskedastic sample) with two fixed effects (one for cross country differences and another considering the time dimension) with changes in international reserves (in logarithm form) as dependent variable and one lag of the change in exchange rates (also in logarithm forms), the volatility of the exchange rate in the previous month, and control (dummies) variables for different markets as independent variables. Table 2.6 presents the results when positive and negative movements of the exchange rate (at month *t-1*), preceding the movements in international reserves, are both considered.[16]

The coefficient found for the exchange rate elasticity is negative and highly significant for all five models which imply that we should expect international reserves to move in the opposite direction of the exchange rate in the previous month. Volatility of the exchange rate also seems to play a role on international reserve swings. The

[16] Later, we consider separately the cases where positive and negative swings in the exchange rate precede the change in international reserves. Separating the analysis into these two cases might be more informative as we previously detected differences in the behavior of the international reserves relative positive and negative changes in exchange rate. By isolating these two different episodes, I am essentially avoiding the omitted variable bias on the estimates.

coefficient for this parameter is positive and significant at the 1% level. Increases in international reserves are associated to increases in the exchange rate volatility. Also, developed countries are less likely to accumulate reserves and developing countries just the opposite. It is easily to see that these results bring support to the fear of floating hypothesis.

Table 2.6: Cross-sectional time-series FGLS regression (panel data: heteroskedastic).
Dependent variable: $Ln(IR_t/IR_{t-1})$

	(1)	(2)	(3)	(4)	(5)
Constant	0.009***	0.012***	0.008***	0.009***	0.010***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$Ln(ER_t/ER_{t-1})$	-0.199***	-0.206***	-0.205***	-0.196***	-0.196***
	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)
ER volatility	0.041***	0.039***	0.039***	0.041***	0.041***
	(0.012)	(0.012)	(0.011)	(0.013)	(0.012)
Developed Markets		-0.004***			
		(0.001)			
Emerging Markets			0.004***		
			(0.001)		
East Asia				0.001	
				(0.001)	
Latin America					-0.001
					(0.001)
Log likelihood	8270.05	8275.58	8274.68	8270.35	8270.53
N. Obs.	6862	6862	6862	6862	6862
Wald χ^2	82.13	94.64	92.76	82.86	83.17
Prob. > χ^2	0.000	0.000	0.000	0.000	0.000

Source: International Financial Statistics; International Monetary Fund.
***, **, *: 99%, 95%, 90% significance level, respectively.

However, based on the previous analyses, we suspect that this last result might be bias since we consider the cases of positive and negative changes in the exchange rate together. To clear this fact, we transform the data into two separate cases isolating these two different episodes. Here we multiply the data by dummy variables corresponding to positive and negative changes in exchange rates. By doing this we isolate the coefficients estimated to each case and have the opportunity to test for differences in the coefficients. The drawback of this approach is that the number of coefficients to be estimated is doubled and so we lose some degrees of freedom. However, because we have a large data set this problem is minimized. Table 2.7 presents the results for the coefficient estimation as well as the t-test for coefficient differences.

As expected, the exchange rate elasticity coefficient is negative and significant at the 1% level for the case of negative or no moment in the exchange rate in the previous month (panel A in table 2.7). There is a tendency to increase reserves after an appreciation of the domestic currency (appreciation avoidance). Also, increases in exchange rate volatility in the presence of exchange rate appreciation seem to influence central banks to increase the level of international reserves. The coefficient, however, reached only 10% significance. Latin American and developed markets increase reserves less comparatively under exchange rate appreciation.

Table 2.7: Cross-sectional time-series FGLS regression (panel data: heteroskedastic) for positive and negative change in exchange rate.
Dependent variable: $Ln(IR_t/IR_{t-1})$

	Coef.	Std. Err.	z	p-value
Panel A: X*Dummy %Δ ERt-1≤0				
Ln(ERt/ERt-1)	-0.287	0.048	-5.920	0.000
ER volatility	0.041	0.023	1.810	0.071
Developed Markets	-0.010	0.005	-1.980	0.047
Emerging Markets	-0.001	0.005	-0.260	0.792
East Asia	0.003	0.002	1.220	0.221
Latin America	-0.011	0.003	-3.640	0.000
Dummy %Δ ERt-1<0	0.017	0.005	3.690	0.000
Panel B: X*Dummy %Δ ERt-1>0				
Ln(ERt/ERt-1)	-0.034	0.039	-0.880	0.380
ER volatility	0.027	0.014	1.960	0.050
Developed Markets	-0.006	0.004	-1.530	0.125
Emerging Markets	0.002	0.004	0.550	0.583
East Asia	0.002	0.002	1.050	0.294
Latin America	-0.002	0.003	-0.580	0.559
Dummy %Δ ERt-1>0	0.007	0.004	1.810	0.070
Log likelihood	8303.51			
N. Obs.	6862			
Wald χ^2	424.16			
Prob. > χ^2	0.000			
t-test (H_0)	**Prob.**	**Accum. Prob.**		
β1 - β8 = 0	0.0000	0.0000		
β2 - β9 = 0	0.5950	0.0002		
β3 - β10 = 0	0.5940	0.0005		
β4 - β11 = 0	0.5833	0.0013		
β5 - β12 = 0	0.8924	0.0030		
β6 - β13 = 0	0.0158	0.0001		
β7 - β14 = 0	0.0924	0.0000		

Source: International Financial Statistics; International Monetary Fund.

As suspected, considering the sub-sample of positive change in the exchange rate at month t-1 (panel B in table 2.7) the exchange rate elasticity is very small and not statistically significant. This result implies that there is no clear evidence that central banks really intervene in the exchange market to contain an eventual further depreciation of the domestic currency. Contradicting even more the fear of floating hypothesis, a depreciation of the domestic currency capable to increase the volatility of the exchange rate induces central banks to buy reserves. The coefficient for the exchange rate volatility is positive and significant at the 5% level.

The *t*-test results for coefficient differences also show that indeed international reserves behave differently after appreciation and depreciation of the domestic currency. It is clear that central banks do not change the level of international reserves in the opposite direction of the exchange rate after depreciation of the domestic currency as they do after appreciation of the domestic currency. Further, increases in the volatility of the exchange rate induce central banks to behave in the same fashion increasing reserves no matter the direction of movement in the exchange rate. The coefficients for exchange rate volatility were not statistically different. Finally, the two models were found statistically different by the *t*-test (total accumulated prob.<0.05). This result shows that estimating the model without separating for the two possible movements in the exchange rate would incur in the omitted variable bias and so the coefficient estimates are not reliable to extract conclusions.

In summary, the results in this paper do not support the fear of floating hypothesis. There is no clear evidence that central banks constantly intervene in the exchange market to control depreciations of the domestic currency. Instead, it is observed a spread tendency among countries to accumulate reserves. A good explanation for this behavior is that during financial crises emerging markets experience shortages of foreign capital and hence high levels of reserves provide insurance against this scenario. Because developing countries face constrained public sector they are leading to hold high stock of international reserves (Aizenman and Marion (2002)). Further, resisting devaluations under adverse economic fundamentals may raise the incentives to devaluate in the future, creating a scenario for speculative attacks (Drazen and Masson (1994); Bartolini and Drazen (1997)). So as the main conclusion of this paper it is appropriate to say that most developing countries are willing to let their currency to float, even because there are benefits they can enjoy, but they do it with a “life jacket”.

2.5. Conclusion

This paper uses a different method to test whether central banks are really constantly intervening in the exchange market as proposed by the fear of floating hypothesis. The parameter used to measure the expected degree of intervention incorporates the probability of an intervention occur and the expected amount used to intervene. Additional analysis evaluates the correlation between international reserves and exchange rates for the two possible occurrences (appreciation and depreciation).

The results show that the main concern of central banks is to accumulate reserves rather than stabilizing the exchange rate. The expected change in international reserves is positive after a depreciation of the exchange rate. This fact is especially true when there is an increase in the exchange rate volatility. Overall, emerging markets show the highest degree of reserve accumulation. The rationale for this behavior relies on the insurance dimension of the sudden-stop problem. Because crises are characterized by foreign currency shortages, there is an incentive for central banks to maintain high levels of international reserves. Besides, by resisting devaluations under adverse economic fundamentals the central bank may create a scenario for a future speculative attack (Drazen and Masson (1994); Bartolini and Drazen (1997)). The implication of these results is that developing countries are in fact willing to let their currency to float but they do it with a “life jacket”. Additionally, the findings in this work suggest that looking at the volatilities of the exchange rate and of reserves, as most studies do, does not provide a good estimate of the willingness of a country to intervene in the foreign market. It is important in this case to consider the sequence of events that characterizes an intervention otherwise the estimate will be exposed to the omitted variable bias leading to misleading conclusions.

This study, however, does not consider the use of interest rate as another channel central banks have to intervene in the exchange market. As suggestion for future research, it would be fruitful to incorporate this aspect together with the use of international reserves applying the same methodology of this paper to calculate the degree of intervention in the exchange market.

3. The Dependence of Mining Activity across Countries on Local Currency Values

3.1. Introduction

Gold mining is obviously a *resource-oriented* activity. In contrast with *market-oriented* service sector activity or other *resource-oriented* activities like farming, however, mining sector activity around the world is extremely volatile. Two volatile financial variables –the world price of gold and local currency exchange rates- may explain why mining is such a volatile industry. This essay poses and tests a dynamic optimal control production/location model of a representative global gold mining company. We model a mining firm's decision about where, when, and by how much to expand gold production in each country given costs in local currency terms, exchange rates, world gold prices, and each country's in-ground gold reserves.

To begin, both gold prices and exchange rates are assumed known in advance with certainty. We show that the optimal extraction path balances revenues with production costs and the "user cost" of depleting the known reserve, in common currency terms, in each country. Then we present the more realistic model which formalizes the real-world uncertainty about gold prices and exchange rate risk. The demand for gold and exchange rates are both assumed to follow a stochastic differential process with a direct dependence on time and space.

Our modeling approach is based on recent versions of the canonical models of resource extraction. The seminal paper on optimal resource exploitation by Hotelling (1931) demonstrated that when the market is competitive and marginal extraction costs are constant, the price over marginal cost premium should rise at the rate of discount. Further, abstracting from demand instability, production will decline monotonically over time to zero. These results offer important insights about why mining activity should be expected to vary over time (and, potentially, space) to maintain an equilibrium flow of the resource. When (or where) the rate of increase in the net price of the resource is below the competitive rate of interest, producers will push production sooner (in order to cash out the resource and invest the proceeds at the higher rate of interest), depleting the resource earlier. On the other hand, when (or where) the net price of the resource grows at a faster rate than the competitive rate of interest, it is optimal to delay extraction to grow the asset "in the ground" rather than in the bond market.

Hotelling anticipated, but did not actually formalize, relevant issues such as the effects of uncertainty and externalities on optimal production dynamics. Peterson and Fisher (1977) and Devarajan and Fisher (1981) provide a thorough review of the literature following Hotelling. Cummings (1969) showed that if costs increase with cumulative production, the optimal rate of extraction is lower than the Hotelling rate, the period of production is thus longer, and the percentage rate of growth in profits is lower than the Hotelling rate as well. Fisher and Krutilla (1975) pointed out that low interest rates tend to stimulate more rapid depletion of natural resources through increase in investment in productive capital.

Schulze (1974) revised the theory to account for entry and exit of mining firms, and examined the impact of internalizing the cost of cumulative environmental damages on the optimal path of resource use. Pindyck (1978) incorporated exploratory activity to offset declining reserves and assumed production costs increase as reserves decline. In this case, producers determine both the optimal levels of exploratory activity and production as they are interrelated. He showed that if the initial reserve endowment is small, the optimal price path will be U-shaped. At first, production increases as reserves are developed, and later, as both exploratory activity and the discovery rate fall, production declines. For large initial reserve endowments, the price will initially increase at a rate below market interest rates, as a consequence of the introduction of exploratory activity, and in the later stages of resource use, price will increase as in the Hotelling model.

Uncertainty was formalized by Pindyck (1980). In his model demand and reserves fluctuate according to continuous-time stochastic processes. He showed that uncertainty has no effect when mining firms are risk-neutral and costs are exogenous and constant, as in Hotelling. However, when extraction costs are endogenous to the level of reserves, reserve uncertainty will affect price. Incorporating risk averse behavior under stochastic output prices, Brennan and Schwartz (1985) show that the decisions to open and close a mine can be evaluated by exploiting the properties of replicating self-financing portfolios.

The economic theory of exhaustible resources has been empirically tested in different ways and settings. For a review of the empirical tests of the theory of exhaustible resources see Chermak and Patrick (2002, 2005). The first systematic empirical examination of natural resource scarcity is attributed to Barnett and Morse (1963) which tested the hypothesis that economic scarcity of natural resource will increase over time in a growing economy. Focusing primarily on extraction costs, measured by the amount of labor and capital needed to produce a unit of output, the authors find that the real cost of five aggregate groups of natural resource commodities declined, rather than increased during the period of 1870 to 1957. Similar results are also obtained using price data. They conclude that the data do not support the hypothesis of increasing resource scarcity. Schulze (1974) argued that the observed increased mineral demand, production, and declining price trends can be explained by decreasing costs of extraction. Adjusting the model to account for technological change, he examines its impact on long run trends in mineral prices. He concluded that "with technological change, market price of minerals may initially decline over much of the time interval of economic use."

This essay presents a theoretical model of the optimal exploitation by a multinational company extracting a nonrenewable and exhaustible resource worldwide.

It elaborates the basic model of Hotelling, as formalized by Pindyck most recently, while extending it to consider alternative locations for extractive activity. The empirical tests rely on data about the global gold sector since 1970, but the model is general enough to apply to any exhaustible resource and time period. Because the profit-maximizing rates of extraction depend on prices, costs, reserve levels, and uncertainties about those determinants, and because those determinants vary across countries over time, mining activity is likely to vary across countries as well as over time.

The contribution of this essay to the literature of exhaustible resources is that the model incorporates three important facts faced by multinational gold mining firms. First, the price of the resource is non-deterministic. Second, firms extracting gold from different countries are exposed to country-specific exchange rate risk. Third, multinational firms choose not only how much to extract, but also from what locations.

The model we pose and test incorporates exchange rate risk explicitly. This is particularly relevant to gold mining because the market price of gold is significantly related to the prices of currencies (exchange rates). In particular, an increase in the world demand for monetary gold causes gold prices to rise in U.S. dollar terms, relative to the (unchanged) dollar cost of extraction in the U.S, increasing the incentives to increase gold extraction activity in the U.S. But the local cost of extraction in other gold producing countries may rise along with world gold prices, depending on how the countries' exchange rates evolve. Thus, the effects of changes in the world price of gold may have magnified effects on the relative levels of activity across countries. The levels of mining activity are not even expected to change in the same direction across all countries with respect to changes in world gold prices.

The remainder of this essay is structured as follows. Section 2 presents the theoretical model of optimal exploitation of an exhaustible resource for multinational companies considering demand for gold and exchange rates are known with certainty. Optimal paths for the resource price and production are analyzed in this section. Section 3 expands the model by assuming the demand for gold and exchange rates follow a stochastic differential process with a direct dependence on time and space. This is a more realistic approach for the problem multinational mining companies face when deciding where, when, and by how much to extract an exhaustible resource. The effects of uncertainty in the resource price and optimal path of production are investigated in this section. Section 4 tests the empirical validity of the theoretical models. Finally, section 5 briefly summarizes the findings.

3.2. The Exhaustible Resource Model with Deterministic Prices and Exchange Rates

This section presents a theoretical model on optimal exploitation for a multinational company extracting a nonrenewable and exhaustible resource worldwide with price and exchange rate being known in advance. The model considers that the exploration activity of the resource relies on a reserve base to be exhausted over time. It assumes therefore no exploration of the resource to accumulate or maintain its level. At each date $t \in [0,T]$, the stock in units of resource remaining in country i at time t, $R_{i,t}$, is given by the initial reserve base, $R_{i,0}$, minus the cumulative resource extractions up to time t. In this context, the transition equation on the resource stock can be written as:

$$R_{i,t} = R_{i,0} - \int_0^t q(s)ds \tag{3.1}$$

The equation above describes the exhaustibility constraint for the resource. The flow of extraction in units of resource for each country, qi,t, is nonnegative at each date t and the initial level of reserves is considered as given. Differentiating the transition equation with respect to time gives the rate of reserve depletion (equation 3.2). Because no exploration is assumed the rate of reserve depletion is entirely defined by the extraction of the resource at each point in time.

$$\dot{R}_{i,t} = -q_{i,t} \tag{3.2}$$

The resource, gold in our case, is priced internationally and the firm has no market power. Production costs, however, are priced locally. Therefore, our first task is to recognize that revenues and costs might be priced in different currencies. In this case, the decision that maximizes profit depends directly on country specific risks such as exchange rate risk.

Assuming that a multinational mining company with base in country j = 1 can extract gold in n different countries (including the company's base country), the optimization problem of this representative miner can be viewed as selecting a path of resource extraction, $Q_t = \{q_{i,t} : i = 1,\ldots,n\}$, where i is the country index, so as to maximize the net present value of profits, i.e.

$$\max_{\{Q_t\}} \int_0^T e^{-rt}\left(\sum_{i=1}^{n} p_t q_{i,t} - x_{i,t} C_{i,t}(R_{i,t}) q_{i,t}\right) dt$$

$$\text{s.t.} \quad \dot{R}_{i,t} = -q_{i,t}, \qquad i = 1,\ldots, n, \quad and \quad t = 1,\ldots, T \tag{3.3}$$

$$R_{i,T} \geq 0, \quad q_{i,t} \geq 0, \quad R_{i,0} = R_i^0 > 0(given) \qquad \forall i$$

where $r > 0$ is the resource extractor discount rate (equal to the competitive interest rate), p_t is the competitive resource price at time t (in U.S. dollars), $x_{i,t}$ is the exchange rate for country i (US$ per domestic currency) at time t, and $C_{i,t}$ is the average cost of production in local currency at time t, which is assumed increasing with the depletion of the reserve base (i.e. $C'_{i,t} < 0 \forall i,t$). All these parameters are taken as given by the firm.

The present-value Hamiltonian of the above problem, denoted by H (t), is:

$$H(t) = e^{-rt}\left[\sum_{i=1}^{n} p_t q_{i,t} - x_{i,t} C_{i,t}(R_{i,t}) q_{i,t}\right] - \left(\sum_{i=1}^{n} \lambda_{i,t} q_{i,t}\right) \tag{3.4}$$

The term in the first brackets measures the total net current cash flow of extracting gold and the last term is the total future losses due to not having the resource. The Hamiltonian yields the following first-order conditions for an interior optimum (i.e. assuming some quantity of gold is extracted in each country at every t):

$$-H_{R_i} = \dot{\lambda}_{i,t} = e^{-rt}\left[x_{i,t}C'(R)q_{i,t}\right], \quad \forall i \tag{3.5}$$

$$H_{q_i} = e^{-rt}\left[p_t - x_{i,t}C(R)\right] - \lambda_{i,t} = 0, \quad \forall i \tag{3.6}$$

$$H_{\lambda_i} = -q_i = \dot{R}_{i,t}, \quad \forall i \tag{3.7}$$

and the transversality conditions

$$\lambda_{i,T^*} \geq 0 \;\; (=0 \;\; if \;\; R^*_{i,T^*} > 0), \;\; \forall i \tag{3.8}$$

We observe that without investment constraint the optimal production decision for each country is totally independent of each other. The first order conditions for the miner's maximization problem are the same for all countries. Assuming (as in Cummings (1969) and Pindyck (1978)) that the marginal cost of extraction increases as reserves are depleted, the change in the present value of future profits resulting from an additional unit of reserves should decrease over time (i.e. $\dot{\lambda}$ is negative). Equation (3.5) represents the dynamic equation for λ, the shadow price of gold. Equation (3.6) describes the static efficient condition. At each point in time, profits are maximized by choosing an extraction path such that the marginal net benefit of the next extracted unit of gold is equal to the value of the loss of that unit from the stock of resource. In other words, the resource user cost at each moment in time should be the discounted difference between price and the marginal cost of extraction. The transversality condition shows that a mining firm will leave resources unextracted at time T^* only if the shadow price is zero.

The equation describing the dynamics of the price path can be obtained by differentiating (3.6) with respect to time, substituting (3.2) for $\dot{R}$ and equating with (3.5).

$$\dot{p} = rp - rxC(R) + \dot{x}C(R) \tag{3.9}$$

Because exchange rates are explicit in this formalization of cost, it is apparent that the optimal extraction price rises more rapidly when the currency in the producing country is appreciating; i.e., when the change in x is positive. On the other hand, when the producing country currency is depreciating, the optimal sale price can rise more slowly than would be predicted by the model that ignores exchange rates. Figure 3.1 illustrates these cases.

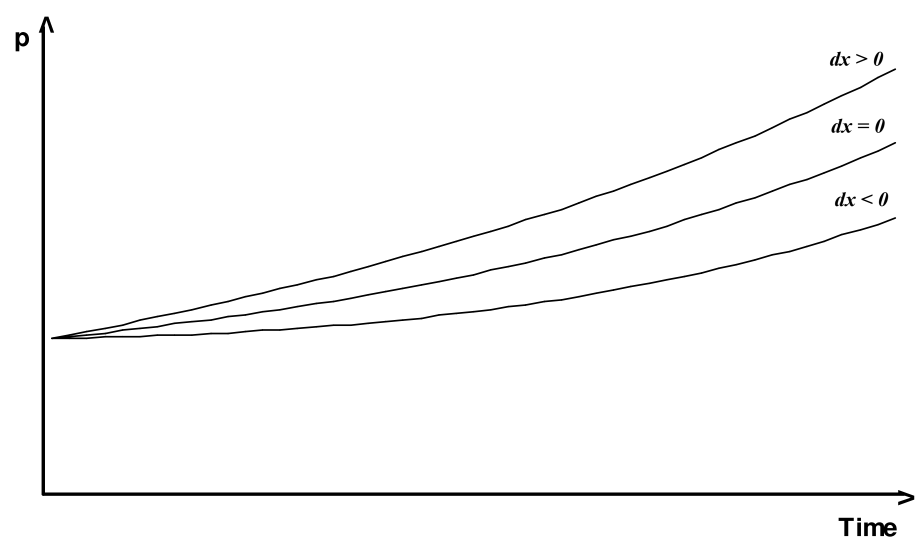


Figure 3.1: Optimal price paths under different exchange rate behavior.

Note also that if there is no exchange rate exposure, i.e., x is always one and $\dot{x}$ is zero; the rate of change of the price path is the same as in the standard constant-cost Hotelling problem and also as in Pindyck (1978) with no dependence of costs on reserves.

In order to parameterize an equation to describe the dynamics of production of gold in a country, we must make some assumptions about demand. Assuming a deterministic downward sloping demand function for gold of the form

$$p = f(q), \tag{3.10}$$

where $f'(q) < 0$, we find the optimal production path by following the same procedure presented above, as:

$$\dot{q} = \frac{r(pq - xqC(R) + f'(q)q) + \dot{x}C(R)}{f''(q)q + 2f'(q)} \tag{3.11}$$

If demand is linear, that is $f''(q) = 0$, since $f'(q) < 0$ we observe that production is increasing when the rate of change in the exchange rate is negative and the cost of extraction reduces in a greater amount than the net benefit of extracting the resource at the production level q. In this case, extracting later increases cash flow due to lower extraction cost in the future when the local currency is more depreciated. However, it is important to note that if the price of the resource is also decreasing over time this could offset the benefit of the reducing cost of extraction, and it may be more beneficial to produce more now than later. Also, if the cost of extraction depends on the level of reserves, increasing production over time might not be optimal even in the presence of domestic currency depreciation as the increase in costs due to lower level of reserves might offset the benefit of a depreciated currency. If demand is non-linear and $f''(q) < 0$, the results described above hold. The difference is that the rate of increase or decrease of production will be smaller. However, if $f''(q) > 0$ the denominator can be either positive or negative depending on the values of the first and second derivatives of the demand function.

3.3. The Exhaustible Resource Model with Stochastic Prices and Exchange Rates

Now consider the case in which the demand for gold and exchange rates are stochastic. When evaluating the extraction choice across many countries, a deterministic setting is unlikely to be appropriate. Uncertainty concerning the exchange rate burdens a risk-averse firm, like an increase in the cost of extraction, in the locality.

In this section, the profit maximization problem of a mining company extracting gold from different countries is formalized. The contribution of this work to the literature of exhaustible resources and the literature on location-activity is that the model incorporates two important facts firms face in this context. First, the price of the resource is non-deterministic. To incorporate this aspect, I assume the demand function for gold follows a geometric Brownian motion. Second, firms extracting gold from different countries are exposed to exchange rate risk which is also incorporated in the model. Exchange rates are assumed to follow a geometric Brownian motion as well.

Consider the demand function for gold, $p(q,t)$, which, to focus on competitive firm choices rather than aggregate market dynamics, is assumed to evolve exogenously over time following the equation

$$p = p(q,t) = y(t)f(q) \tag{3.12}$$

where $f(q)$ is exogenous or parametric (due to competitiveness), $f'(q) < 0$ (demand is downward sloping), and $y(t)$ follows a Wiener process of the form

$$\frac{dy}{y} = \alpha_1 dt + \sigma_1 dz_1 . \tag{3.13}$$

Equation (3.13) states that, over a short time interval, the proportionate change in the stochastic term in the demand for gold is normally distributed with mean $\alpha_1 dt$ and variance $\sigma_1^2 dt$. In this case, α_1 represents the trend of the demand for gold (the percentage "drift") and σ_1 represents the rate of rotation of the demand curve (the percentage volatility or "diffusion"). Setting the demand function in this way allow us to incorporate the fact that at time zero the demand is known with certainty, but that uncertainty about its rate of growth increases with the time horizon. Additionally, fluctuations in demand occur continuously over time.

Embedding all the relevant information at time *t*, the expected instantaneous value of *y* and its variance is given by:

$$E_t(dy) = y\alpha_1 dt \quad (3.14)$$

$$E_t(dy)^2 = y^2\sigma_1^2 dt. \quad (3.15)$$

Furthermore, the term dz_1 in equation (3.13) represents the random increments to the Wiener process z_1. The increments are independent at any instant and satisfy the following conditions:

$$Edz_1 = 0 \quad (3.16)$$

$$E(dz_1^2) = dt. \quad (3.17)$$

The exchange rate, x, also evolves exogenously over time as geometric Brownian motion. In this case, the dynamics of the exchange rate can be expressed as a linear stochastic differential equation

$$dx = x\alpha_2 dt + x\sigma_2 dz_2 \quad (3.18)$$

where the term $x\alpha_2$ represents the expected instantaneous variation in the exchange rate over time (α_2 is the expected rate of change in the exchange rate) and $x\sigma_2$ measures the instantaneous standard deviation of the exchange rate (σ_2^2 is the per-unit-time variance of the rate of change in the exchange rate). The dynamics of the exchange rate equation shares the same characteristics as for the demand equation.

Finally, ρ_{yx} is the instantaneous coefficient of correlation between the Wiener processes, dz_1 and dz_2. For some countries, we can assume that this correlation is zero – that is there is no correlation between the price of gold and the exchange rate – however, for others that assumption is clearly testable. So, the model allows for the possibility that the correlation between the price of gold and the producing country exchange rate might be different than zero, and may be different across countries.

Having defined the price of gold and exchange rates, we can now turn to the problem that a representative mining company will face when choosing where to extract gold. Consider a risk-neutral firm that maximizes profit in American dollar terms. This problem can be posed as follows:

$$\begin{aligned}
&\max_{\{Q_t\}} E_0 \int_0^T e^{-rt} \left(\sum_{i=1}^{n} y_t f(q_t) q_{i,t} - x_{i,t} C_{i,t}(R_{i,t}) q_{i,t} \right) dt \\
&\text{s.t.} \quad dR_i = -q_i dt, \qquad i = 1,\ldots, n, \quad and \quad t = 1,\ldots, T \\
&\qquad \frac{dy}{y} = \alpha_1 dt + \sigma_1 dz_1 \\
&\qquad \frac{dx_i}{x_i} = \alpha_{2,i} dt + \sigma_{2,i} dz, \quad \forall i \\
&\qquad R_{i,T} \geq 0, \quad q_{i,t} \geq 0, \quad R_{i,0} = R_i^0 > 0 (given) \qquad \forall i
\end{aligned} \tag{3.19}$$

Abstracting from a constraint on overall firm size, the optimal levels of activity across locations are independent. The multinational firm chooses optimal *q* paths (i.e., the profit-maximizing level of activity) in each possible location independently. To solve the stochastic dynamic problem in any one country, first define the optimal value function, J, which is a function of y, x, R, and t.

$$J = J(y,x,R,t) = \max E_t \int_t^T \pi_d(\tau) d\tau \tag{3.20}$$

The fundamental equation of optimality is

$$0 = \max_q \left[\pi_d(t) + (1/dt) E_t dJ \right] \tag{3.21}$$

Expanding the term containing the differential operator $(1/dt)E_t d(\cdot)$, provide us with the following objective function:

$$\phi(q; y,x,R,t) = \pi_d(t) + J_t - qJ_R + \alpha_1 y J_y + \alpha_2 x J_x + \frac{1}{2} y^2 \sigma_1^2 J_{yy} + \frac{1}{2} x^2 \sigma_2^2 J_{xx} + yx\rho_{yx}\sigma_1\sigma_2 J_{yx}$$

(3.22)

The derivative with respect to q gives

$$\frac{\partial \phi}{\partial q} = \frac{\partial \pi_d}{\partial q} - J_R = 0 \tag{3.23}$$

In order to obtain an empirically tractable hypothesis, we eliminate J from the problem. This can be accomplished by first differentiating equation (3.22) with respect to R

$$\frac{\partial \pi_d}{\partial R} + J_{Rt} - qJ_{RR} + \alpha_1 y J_{Ry} + \alpha_2 x J_{Rx} + \frac{1}{2} y^2 \sigma_1^2 J_{Ryy} + \frac{1}{2} x^2 \sigma_2^2 J_{Rxx} + yx\rho_{yx}\sigma_1\sigma_2 J_{Ryx} = 0 \tag{3.24}$$

and then using Ito's lemma to rewrite the equation as

$$\frac{\partial \pi_d(t)}{\partial R} + (1/dt)E_t dJ_R = 0. \tag{3.25}$$

Now, we can apply the differential operator $(1/dt)E_t d(\cdot)$ to equation (3.23) to get

$$(1/dt)E_t d(\partial \pi_d / \partial q) = (1/dt)E_t dJ_R. \tag{3.26}$$

Combining equations (3.25) and (3.26) we find

$$(1/dt)E_t d(\partial \pi_d / \partial q) = -\frac{\partial \pi_d}{\partial R} \tag{3.27}$$

Assuming the market of gold is competitive, we know that

$$\frac{\partial \pi_d}{\partial q} = \frac{\pi_d}{q} = [p(q,t) - xC(R)]e^{-rt}, \tag{3.28}$$

which substituted into (3.27) gives

$$-r[p(q,t) - xC(R)] + [(1/dt)E_t dp - C(R)(1/dt)E_t dx - x(1/dt)E_t dC(R)] = xqC'(R). \tag{3.29}$$

Now let expand $dC(R)$ using Ito's lemma

$$dC(R) = C'(R)dR + \frac{1}{2}C''(R)(dR)^2 \tag{3.30}$$

Substituting in (3.30) into (3.29) and knowing that $E_t(dR)^2 = 0$, we find the equation for the expected price dynamics of gold

$$(1/dt)E_t dp = r[p(q,t) - xC(R)] + C(R)\alpha_2 x. \tag{3.31}$$

Noting that $(1/dt)E_t dx = \alpha_2 x,$ we observe that the expected instantaneous rate of change of the competitive price under stochastic exchange rate is exactly the same as in the certainty case. This is because in our model uncertainty is incorporated only with respect to future values since the current level of the price of gold and the exchange rate are known exactly at each moment of time. This feature of the model causes the expected rate of change of the value of the resource to be unaffected by uncertainty.[17] Additionally, we observe that if the expected rate of change in the exchange rate is positive (domestic currency depreciating) price will begin lower and will rise more rapidly. However, under negative expected rate of change in the exchange rate (domestic currency appreciating) the price will be higher initially but will rise more slowly.

Now, to obtain the expected dynamics of production of gold in any one particular country, first remember that $q = q^*(y, x, R)$ along the optimal trajectory. Expanding the differential dq using Ito's lemma:

$$dq = q_y dy - q_x dx + q_R dR + \frac{1}{2} q_{yy} (dy)^2 + \frac{1}{2} q_{xx} (dx)^2 + \frac{1}{2} q_{RR} (dR)^2 + q_{yx} dydx + q_{yR} dydR + q_{xR} dxdR$$

Using the definitions of dy and dx (equations 3.14 and 3.19, respectively), we obtain

$$E_t\left[(dq)^2\right] = q_y^2 y^2 \sigma_1^2 dt + q_x^2 x^2 \sigma_2^2 dt + 2 q_y q_x \sigma_1 \sigma_2 xy \rho_{yx} dt + o(t) \tag{3.32}$$

and

$$E_t\left(dqdy\right) = q_y y^2 \sigma_1^2 dt + q_x \sigma_1 \sigma_2 xy \rho_{yx} dt + o(t) \tag{3.33}$$

where, in both equations, $o(t)$ represents terms that vanish as $dt \to 0$.

[17] Pindyck (1980) provides a detailed discussion of the difference in nature of uncertainties in this kind of setting and in others studies such as Loury (1978) and Gilbert (1979) which additionally incorporate "current" uncertainty.

We can now use Ito's lemma on the dynamics of the price of the resource which give us

$$dp = yf'(q)dq + f(q)dy + \frac{1}{2}yf''(q)(dq)^2 + f'(q)dqdy. \tag{3.34}$$

Applying the differential operator $(1/dt)E_t d(\cdot)$ in both sides of equation (3.34) we get

$$\begin{aligned}(1/dt)E_t dp = yf'(q)(1/dt)E_t dq + f(q)(1/dt)E_t dy + \frac{1}{2}yf''(q)(1/dt)E_t(dq)^2 \\ + f'(q)(1/dt)E_t(dqdy)\end{aligned} \tag{3.35}$$

Finally, substituting in (3.31), (3.32), and (3.33) into equation (3.35) and solving for $(1/\partial t)E_t dq$ give us the expected production dynamics of gold in one particular country:

$$(1/dt)E_t dq = \frac{r(p - xC(R)) - (\alpha_1 p - \alpha_2 xC(R)) - \gamma(\sigma_1, \sigma_2, \rho_{yx})}{yf'(q)} \tag{3.36}$$

where

$$\gamma(\sigma_1, \sigma_2, \rho_{yx}) = f'(q)(q_y y\ \sigma_1^2 + q_x yx\sigma_1\sigma_2\rho_{yx}) + \frac{1}{2}yf''(q)(q_y^2 y^2\sigma_1^2 + q_x^2 x^2\sigma_2^2 + 2q_y q_x yx\sigma_1\sigma_2\rho_{yx})$$

First let analyze the sign of $\gamma(\sigma_1, \sigma_2, \rho_{yx})$, an important component of the dynamics of production equation. We observe that the sign of $\gamma(\sigma_1, \sigma_2, \rho_{yx})$ depends in part on the sign of q_y. We know that an increase in *y* causes the demand curve to rotate to the right. Since prices increase, we should expect production also to increase under this scenario. This leads to the conclusion that $q_y \geq 0$. We have also seen that increases in *x* cause prices to rise more rapidly. Therefore, q_x should also be nonpositive. Now, assuming linear demand function and no correlation between resource price and exchange rate, it is easy to see that a more volatile resource price will cause producers to be more conservative about the future. This will lead to a higher initial production level and lower present price of the resource. In the presence of negative correlation between the resource price and exchange rate, increases in volatility lead producers to act in an even more conservationist way. This result is expected since negative correlation between these variables amplifies producers' gains/losses. If $\rho_{yx} > 0$, the net effect is undetermined.

Now let consider the case where the second derivative of the demand function is nonzero. In this case, if $f''(q) \leq 0$ and $\rho_{yx} < 0$, $\gamma(\sigma_1, \sigma_2, \rho_{yx})$ will be more negative, inducing a faster reduction in production. However, when the correlation is positive, $\gamma(\sigma_1, \sigma_2, \rho_{yx})$ is of undetermined sign no matter the sign of $f''(q)$. In this case, the net effect in production will depend on the extent of the correlation between these variables and their corresponding volatility.

Finally, equation (3.36) shows that the expected production dynamics of gold in one particular country depends directly on the opportunity cost of extracting the resource in any particular time, the expected instantaneous marginal benefit or loss of the next unit in the ground and the degree of uncertainty of the resource future price and its cost of extraction.

3.4. Empirical Results

In order to observe if the theoretical hypotheses are reflected in the actual data of gold production around the world, we conduct an empirical analysis to study the relationship between the level of gold production and the international price of gold and the exchange rate for major gold producer countries over the last decades.

The data consists of annual gold production and exchange rates of 29 countries from all parts of the world including Africa, Asia, Oceania, Europe, North America, and Latin America.[18] Countries included are Australia, Bolivia, Brazil, Canada, Chile, China, Colombia, Congo/Zaire, Ecuador, Ethiopia, Fiji, Ghana, Honduras, India, Indonesia, Japan, Korea, Mexico, New Zealand, Nicaragua, North Korea, Papua New Guinea, Peru, Philippines, South Africa, Sweden, Tanzania, United States, and Venezuela. The period of analysis ranges from 1970 to 2005. Exchange rates and price of gold are taken annually in terms of local currencies versus United States dollars.

Consistent with the model outlined in the previous section, table 1 presents some estimated parameters about the dependence of the production of gold to the movements

[18] Data sources are the International Monetary Fund for the price of gold and the exchange rates and the United States Geological Survey for the production of gold.

in the international price of gold and the exchange rate. The objective is to investigate whether mining activity across countries is affected by local currency values. The method of estimation is the random-effects generalized least square (GLS) regression on the time series for the 29 countries.

The estimated equations are:

$$\ln q_{i,t} = \alpha + \beta_1 \ln x_{i,t} + \beta_2 \mu_{i,t} + \beta_3 \ln p_t + \beta_4 \ln p_{t-1} + \beta_5 \ln p_{t-2} + \beta_6 \ln p_{t-3} + \beta_7 \sigma_{P_t} + \beta_8 \sigma_{P_{t-1}} + u_i + \varepsilon_{i,t} \quad (3.37)$$

$$\varepsilon_{i,t} = \rho \varepsilon_{i,t-1} + z_{i,t}$$

(3.38)

where $q_{i,t}$ is the annual gold production, $x_{i,t}$ is the end-of-the-year exchange rate (US$/local currency), p_t is the end-of-the-year price of gold (also taken for different lags), $\mu_{i,t}$ is the coefficient of variation of the exchange rate,[19] σ_{P_t} is the volatility of the price of gold (monthly standard error for the year), u_i is a country specific random element, and $\varepsilon_{i,t}$ is the disturbance term which is considered first-order autoregressive.

The empirical results of our model is presented in table 3.1. The coefficient found for the exchange rate parameter is negative and significant at the 10% level, which implies that the production of gold decreases in years that the local cost of extraction

[19] The coefficient of variation of the exchange rate was calculated by dividing the monthly standard deviation of the exchange for the year by the monthly average quotation of the exchange rate.

increases due to exchange rate movement, as hypothesized. Volatility of the exchange rate also seems to play a role on international gold production swings. The coefficient for this parameter is negative and significant at the 5% level, also as hypothesized. Decreases in production of gold are associated with increases in-exchange rate volatility.

Table 1: The determinants of gold production around the world.
Random-effects GLS regression (panel data).
Dependent variable: $Ln(q_t)$

	Coefficient	Std. Error	z	p>|z|
Constant	9.949	0.624	15.930	0.000
$Ln(x_t)$	0.037	0.020	1.890	0.059
μ_t	-0.315	0.126	-2.500	0.012
$Ln(p_t)$	0.218	0.114	1.900	0.057
$Ln(p_{t-1})$	0.196	0.098	2.000	0.045
$Ln(p_{t-2})$	-0.099	0.096	-1.030	0.305
$Ln(p_{t-3})$	0.185	0.085	2.170	0.030
$\sigma_{p\,t}$	-0.003	0.002	-2.190	0.029
$\sigma_{p\,t-1}$	-0.006	0.002	-3.570	0.000
ρ	0.903			
σ_u	1.540			
σ_e	0.510			

N. Obs. = 1044
Wald χ^2 (11) = 45.84
Prob. > χ^2 = 0.000
R-sq: Within = 0.2691
Between = 0.0089
Overall = 0.0558

Sources: International Financial Statistics; International Monetary Fund.
Minerals – Statistics and Information; United States Geological Survey

The price of gold shows both short term and long term positive effects on international production of gold. Risks associated with the value of the mineral also impact negatively its production around the world. The coefficients for the price volatility in the current year and in the previous year are negative and significant at the 5 and 1%

level, respectively. Higher local currency cost induces a more conservationist behavior of producers. Also, it is worth to mention that the model was able to capture a large and positive autocorrelation of the error terms indicating that inertia in countries' production of gold.

In sum, the results presented in this section support our theoretical hypotheses. The price of gold and exchange rates are indeed determinants of international production of gold across countries, and accounting for their movements helps explain the levels of extraction by mining companies around the world.

3.5. Conclusion

This paper develops a dynamic optimal control production/location model of a representative global gold mining company. The firm must decide where, when, and by how much to expand gold production in each country given each country's in-ground gold reserves. The main contribution of this paper to the literature of exhaustible resources is that it incorporates the role the exchange rate risk imposes on the production decision process when operating in a foreign country. The contribution to the theory of the location of production is that it formalizes an extractive firm's dynamic choice of how much to extract within a context of currency costs and risks that vary across alternative locations.

In the first model, both gold prices and exchange rates are assumed known in advance with certainty. The results show that when the local currency is appreciating (depreciating) the resource price rises more rapidly (slowly). Furthermore, in the absence of exchange rate exposure, the rate of change of the price path is the same as in the standard constant-cost Hotelling problem. The results also demonstrate that the optimal extraction path balances revenues with production costs and the "user cost" of depleting the known reserve, in common currency terms, in each country. Changes in the exchange rate can significantly affect total costs of extraction. We observe that the exchange rate appears to have a strong impact on the production level over time.

The more realistic model formalizes the real-world uncertainty about gold prices and exchange rate risk. In this model, the demand for gold and exchange rates follow a stochastic differential process with a direct dependence on time and space. The resulting hypothesis is that the expected instantaneous rate of change of the competitive price under stochastic exchange rate is exactly the same as in the certainty case. This is because in the model uncertainty is incorporated only with respect to future values since the current level of the stochastic variables are known exactly at each moment of time. In terms of optimal production path, the correlation between the resource price and the exchange rate and their volatility can determine the decision to expand or contract the rate of resource extraction over time. For example, assuming linear demand and no correlation between resource price and exchange rate, a more volatile resource price will cause a producer to be more conservative about the future. This leads to an increase in the rate at which production is expected to be reduced over time, leading to a higher initial production level and a lower present price of the resource. In the presence of negative correlation between the resource price and exchange rate, which amplifies producers' gains/losses, increases in volatility lead producers to act in an even more conservationist way. When the correlation between the stochastic variables is positive, the change in future production is of undetermined sign. In this case, the net effect on production will depend on the extent of the correlation between these variables and their corresponding volatility.

Finally, our empirical results support the theoretical models posed here. The level and volatility of the price of gold and the levels and volatilities in exchange rates are indeed determinants of production of gold across countries around the world.

4. Incorporating Market Trend into the Volatility-Weighted Historical Simulation Approach for Anticipating Market Risk

4.1. Introduction

Since Value-at-Risk (henceforth VaR) was developed[20], it has become a standard tool of risk measurement among financial institutions. The idea behind the technique is that, given a time horizon and confidence level, VaR estimates the maximum loss that can possibly occur. It is essentially a static model (one-period model) in which positions remain unchanged over the risk horizon. The appeal of the VaR measure relies on the fact that it aggregates several components of risk into one single number. Additionally, as pointed out by Penza and Bansal (2001), the technique is flexible in the sense that by only choosing the appropriate period of time and probability level, the tool can be adapted to institutions' specific needs.

One general shortcoming of VaR approaches is that different models describing the same market can result in different risk estimates. For instance, Beder (1995) shows

[20] See JP Morgan (1994, 1997) and Phelan (1997).

that different results for same portfolios can occur depending on which VaR technique is adopted. Furthermore, the results reveal that the VaR measure is highly dependent on parameters, data, and assumptions.

But perhaps the most serious conceptual problem with VaR is that it disregards any loss beyond its critical value. This problem, called the tail risk, was address by Artzner et al. (1997) and Embrechts et al. (1998). Being aware of this fact is so critical that, as shown by Yamai and Yoshiba (2005), in certain real-world cases investors may take wrong decisions based on VaR. The example presented by the authors is an investor who assembles his portfolio without taking into account the magnitude of unlikely losses (losses beyond the VaR critical value) that eventually may occur. The authors also show that in high volatile markets or in markets in which the assets have extreme dependence structure, VaR may underestimate risk. Further, using a dynamic portfolio optimization framework, Basak and Shapiro (2001) show that an investor who wants to maximize his utility under VaR constraint could result in choosing a position that can lead to losses beyond the VaR critical value. Additionally, Yamai and Yoshiba (2002) present the same problem for the case of a concentrated credit portfolio and far-out-of-the-money option.

To circumvent the main weakness of the VaR technique, Rockafellar and Uryasev (2000, 2002) suggest the Conditional Value-at-Risk, or CVaR,[21] which takes into account losses outside the VaR quantile. Supported by the extreme value theory (EVT), CVaR

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[21] This risk measure is also called in the literature as expected tail loss, expected shortfall, tail VaR, tail conditional expectation and worst conditional expectation.

represents the conditional expectation of loss (mean loss) that is beyond the VaR level. It basically indicates what we can expect to lose given the loss is beyond VaR. As can be noted, CVaR will always produce a risk measure that exceeds VaR. As pointed by Dowd (2002), CVaR has the many attractions of the VaR measure and, further, it has also some additional advantages: it tells the analyst what to expect in bad states (tail events); it does not discourage risk diversification as VaR sometimes does; and CVaR estimates are less prone to sampling error than VaR.

Although CVaR has many advantages over VaR, the truth is that these risk measure techniques complement each other. VaR informs investors about ordinary loses while CVaR focus on unusual loses. Therefore, these methods are not exclusives but rather should be used together to provide a more complete risk estimate. Additionally, one should note that to estimate CVaR we first need to estimate VaR. Consequently, the quality of the CVaR estimate is closely related to the quality of the VaR estimate.

In this study, we incorporate to the well known volatility-weighted historical simulation approach an additional parameter to improve VaR and CVaR estimation for different assets. Fundamentally, this new parameter incorporates the presence of market trends into the risk estimate to avoid overestimating or underestimating risk under these circumstances. The idea is to correct VaR and CVaR measures when there is a tendency (bullish or bearish) in the market. The model is tested in the context of daily asset returns by applying the binomial test on failure rates. Additionally, the CVaR measures are compared to the losses exceeding VaR through the sample differences paired test and their correlation. The results show that accounting for market trends can improve VaR estimates for different assets, especially extreme VaRs (1% and 99%-VaRs). The contribution of the trend parameter in estimating CVaRs however is not evident.

The remainder of this essay is structured as follows. In the next section, formal definitions of VaR and CVaR measurements are presented. Section 3 introduces the methodology used in this study. Section 4 reports the results of the tests applied. Section 5 briefly summarizes our findings.

4.2. Risk Measures

There are different approaches to calculate VaR and, consequently, CVaR.[22] All forecast risk by analyzing historical patterns of market variables. Basically, they can be classified into two broad categories: parametric approaches and non-parametric approaches.

Parametric approaches seek to estimate VaR and CVaR measures based on assumptions about the distribution of the parameter of interest (for example, profits/losses or asset returns). In this case, it is necessary to explicitly specify the distribution from which the observations are drawn. The normality assumption is the most common used in

[22] See Duffie and Pan (1997), Crouhy, Galai and Mark (2001), Jorion (2001) and Gallati (2003) for an overview.

parametric approaches. Nevertheless, when the joint distribution of the data is non-elliptical, the quantile-based VaR may not behave as a coherent risk measure. More specifically, it may not satisfy the subadditive property.[23] In this case, VaR of a combined position may be greater than the sum of VaRs of the positions considered individually.

To circumvent the problem of the non-subadditivity property, Rockafellar and Uryasev (2000, 2002) suggest a different risk measure, called Conditional VaR (CVaR). The CVaR parameter measures the magnitude of potential losses in the tail by determining the expected extreme loss with a predefined confidence level. It is the conditional expectation of loss given that the loss is beyond the VaR level. The CVaR is calculated as the average of the tail VaRs at a chosen confidence level. It is the probability-weighted average of tail losses, or losses exceeding VaR. The CVaR measure satisfies all proprieties of a coherent risk measure. So, now aggregating individual risks does not increase overall risk. However, estimating CVaR is computationally more intensive and sometimes its estimation can be significantly more difficult.

[23] According to Artzner et al. (1997, 1999), in order to be coherent, any risk measure should establish a correspondence ρ between the space X of random variables and a non-negative real number - $\rho : X \rightarrow R$ (see also Frittelli and Gianin (2002), Szegö (2002), and Cheng et al. (2004)). Scalar measures of risk allow to order and compare positions according to their respective risk value. Therefore, ρ must satisfy the following properties:

(i) Positive homogeneity: $\rho(\lambda x) = \lambda\, \rho(x)$ for all random variables x and all positive real numbers λ;
(ii) Subadditivity: $\rho(x + y) \leq \rho(x) + \rho(y)$ for all random variables x and y (it is easy to note that any positively homogeneous functional ρ is convex if and only it is subadditive);
(iii) Monotonicity: $x \leq y$ implies $\rho(x) \leq \rho(y)$ for all random variables x and y; and
(iv) Transitional invariance: $\rho(x + \alpha r_0) = \rho(x) - \alpha$ for all random variables x and real numbers α, and all riskless rates r_0.

Non-parametric approaches do not require making assumptions about the distribution of the data. VaR and CVaR measures are obtained directly from the empirical distribution. All non-parametric approaches assume that the near future risk will look sufficiently like the past so it can be forecasted using data from a precedent period. The most popular non-parametric approach is the historical simulation.[24]

Estimating VaR using the historical simulation technique consists simply in taking the observation from the frequency histogram drawn from the empirical data. The observation that cuts off the lower one minus the confidence level (i.e.,1-*cl*) of very high losses from the rest of the distribution determines the VaR measure. The CVaR is obtained by taking the average of the tail VaRs.

One important aspect of this kind of approach is the choice of the length of the data period used to estimate the VaR and CVaR measures. Choosing large periods for estimation gives raise the problem of aged data. On the other hand, choosing small periods may lead to imprecise VaR estimations and, sometimes, the CVaR estimate cannot be achieved. Also, if the past data is equally weighted, unlikely to recur past events will influence the estimation of VaR until they get old enough to be excluded. This fact, called ghost effects, causes the estimation to be less responsive to current market conditions.

[24] There are others non-parametric approaches used to estimate VaR and CVaR as bootstrap methods, non-parametric density estimation methods, and principal components and factor analysis methods.

One way to reduce the ghost effects is to modify the simple historical simulation method by given weights to the past data (Boudoukh, et al. (1998)). Age weighting treats most recent observations as having more important information to forecast the near future risk. However, as pointed by Hull and White (1998), this method reduces the effective sample size. In addition, Pritsker (2001) shows that VaR estimates using age weighting can still be insufficiently responsive to changes in the underlying risk.

An alternative way to weight the data, suggested by Hull and White (1998), is to use volatility. Volatility-weighted historical simulation uses recent changes in volatility to update return information. This approach estimates VaR by replacing the returns in the data set, $r_{t,i}$, with volatility-adjusted returns, $r^*_{t,i}$. VaR and CVaR parameters are then found in the usual way. The volatility-adjusted returns are calculated as follows:

$$r^*_{t,i} = \frac{\sigma_{T,i} \cdot r_{t,i}}{\sigma_{t,i}} \tag{4.1}$$

where $r_{t,i}$ is the historical return on asset i on day t in the historical sample, $\sigma_{t,i}$ is a historical forecast measure of the volatility of the return on asset i on day t, made at the end of day t-1, and $\sigma_{T,i}$ is the most recent forecast of the volatility of asset i. In this formula, if the current forecast of volatility is greater than the estimated volatility of period t, then actual returns in any period t will be higher. Therefore, actual returns in any period T are increased (or decreased) depending on whether the current forecast of volatility is greater (or less than) the estimated volatility for period t.[25]

The volatility-weighted historical simulation approach produces risk estimates that are sensitive to volatility changes. Further, it can produce VaR and CVaR estimates greater than the maximum loss in the data set in high volatile periods. So this approach eliminates limit in VaR and CVaR estimates imposed by standard historical simulation approaches in which the maximum future loss cannot exceed the maximum past loss. Finally, Hull and White (1998) present empirical evidences that the volatility-weighted historical simulation approach produces superior VaR estimates relative to approaches that do not take into account changes in volatility.

This essay seeks to incorporate one important additional parameter on the volatility-weighted historical simulation technique which accounts for the presence of trends in the market. Accounting for trends can avoid overestimation and underestimation of VaR and CVaR measures in both sides of the trade (that is for long and short positions). The new parameter is easily incorporated into the above equation for the volatility-weighted historical simulation approach. The volatility and trend-adjusted returns can be calculated according to the equation:

$$r_{t,i}^{*} = \frac{\sigma_{T,i} \cdot (r_{t,i} + \beta_{t,i})}{\sigma_{t,i}} \tag{4.2}$$

[25] Dowd (2002, p.68)

where all parameters are the same as before except the additional parameter $\beta_{t,i}$ which represents market trend at time t for asset i. It can be measured by the slope of the regression on exponentially smoothed returns. In this formula, volatility and market trend affect the modified returns from which VaR and, consequently, CVaR are calculated.

The advantage of this approach is that it will only modify the estimation provided by the volatility-weighted historical simulation when a trend is detected in the market. Besides, this approach allows incorporating into the risk estimation changes in the market sentiment more quickly. If current market is bullish (bearish), all returns used for estimation will be corrected to the upside (downside). This is an important aspect that none of previous models have considered. Unless recent observations are tail observations, VaR and CVaR estimates from previous historical simulation approaches will not change considering the nature of the observations (positive or negative returns). Essentially, the risk estimation process of our model not only includes recognizing changes in volatility but also detecting the presence of market trends.

4.3. Method

We seek to compare the applicability of different approaches in measuring market risk for different types of assets. Particularly, we are interested in investigating the gain of accounting for market trends in estimating risk by means of VaR and CVaR. In this study, we apply volatility-weighted historical simulation with and without our market trend parameter using GARCH and EWMA to estimate volatility. Therefore, we test and compare four models.

In order to apply the volatility-weighted historical simulation method, however, it is necessary to forecast volatility of the asset. The first model used to forecast volatility is the largely used GARCH (Bollerslev (1986) and Taylor (1986)). This method is very efficient in identifying volatility clusters in time series data. It assumes that the conditional variance is governed by a linear autoregressive process of past squared returns and variances. Considering only one past period, which is the most common used (Poon and Grager (2003)), the volatility estimates using GARCH (1,1) model follow the equations:

$$r_t = \gamma_0 + \gamma_1 r_{t-1} + e_t, \tag{4.3}$$

$$e_t \mid I_{t-1} \sim N(0, \sigma_t^2), \tag{4.4}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 \sigma_{t-1}^2, \tag{4.5}$$

where I_{t-1} denotes time series history up to time *t-1*. In this model, the restrictions α_0, α_1, $\alpha_2 \geq 0$ ensure that the conditional variance is positive and stationarity in variance imposes that $\alpha_1 + \alpha_2 < 1$.

Another approach used in this study to forecast volatility is the exponentially weighted moving average (EWMA). The estimates of the volatility using the EWMA model are given by the approximation following the equation:[26]

$$\sigma_t^2 \approx \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 \tag{4.6}$$

where the estimate, σ_t, of the volatility for day *t* (made at the end of day *t-1*) is obtained from σ_{t-1} (the estimate from previous day of the volatility for day *t-1*) and r_{t-1} (the most recent observation on changes in the daily return).

This approach places more weight on recent observations when forecasting volatility, reducing ghost effects and, at the same time, capturing volatility clusters. It basically assumes that recent observations are more important in forecasting the near future risk than old observations.

The volatility-weighted historical simulation approach consists in updating return information by taking account recent changes in volatility. Returns used to estimate VaR and CVaR become volatility-adjusted by using equation 4.1 (presented in the previous section).

After obtaining the volatility estimates through the methods above (GARCH and EWMA), we proceed by calculating the volatility-adjusted returns with and without the market trend parameter which is defined as the slope of the OLS regression of the last 5 exponentially smoothed observations. The data is smoothed using also the EWMA

[26] The derivation of the equation characterizing the EWMA model is presented in Appendix A.

process in which lambdas are defined to provide the best fit of the model. After transforming the returns, we easily find our VaR estimates by assembling the histogram of frequency and taking the observation that cuts off the lower one minus the chosen confidence level of very high losses from the rest of the distribution. In this study, we use 200 past observations to calculate VaR and CVaR measures for the 99%, 95%, 5%, and 1% confidence levels. Therefore, in our case, the 3^{rd} , 11^{th} , 190^{th} , and 198^{th} smallest return in the sample corresponds to our 99%, 95%, 5%, and 1% VaR, respectively. CVaR is obtained by simply taking the average of the tail VaRs.

The sample used is the daily percent change of exchange rates (Euro/US$, Yen/US$, and Brazilian Real/US$), financial assets (S&P500 and Brazilian Ibovespa Index), metals (Gold and Silver), and energies assets (Crude Oil and Heating Oil).[27] The models are tested starting in the first day of trade of 2001 through the last day of trade of 2007. The VaR values are updated daily and the data used for estimation also moves accordingly. Then each actual daily percent change is compared to the daily VaR measure, giving a frequency of tail losses.

To give support and legitimacy to our VaR results, for each model, we apply the binomial test (Kupiec (1995)) over the failure rates, which are given by the proportion of times VaR is exceeded. Essentially, this test evaluates whether the number of observed

[27] For all assets we use the close value of the day. Gold refers to the London PM Fix, Silver also refers to the London PM fix, Crude Oil refers to Cushing OK WTI Spot Price FOB, and Heating Oil refers to New York Harbor No. 2 Heating Oil Spot Price FOB.

exceptions is in accordance with the hypothesized values. In order to implement Kupiec's test, we first modify the observations to a binomial framework. In this case, each observation exceeding VaR takes form of one and non-exceeding observations becomes zero. The sequence of failures *x* follows a binomial probability distribution given by:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \tag{4.7}$$

where *n* is the number of observations and *p* is the predicted frequency of tail losses. Therefore, the expected value of *x* is *pn* and its variance is given by *pn(1-p)*.

For *n* large enough, by the central limit theorem, the binomial distribution can be approximated to the normal distribution. Then follows that:

$$z = \frac{x - pn}{\sqrt{p(1-p)n}} \approx N(0,1) \tag{4.8}$$

The hypothesis of interest is whether the model provides good prediction about the number of losses or gains that one should expect to exceed VaR measures.

However, the binomial test only focuses in the frequency of the tail losses. The sizes of the losses are not evaluated by this test. Nevertheless, the VaR measure does not attempt to estimate the loss when it is beyond its value. This is out off its scope. On the other hand, the CVaR measure gives the expected loss given the loss is greater than VaR. Therefore, this measure is appropriate to be included in the test involving the size of the losses.

The test used to evaluate the effectiveness of the CVaR measure in predicting the value of the losses when these are greater than VaR was the *t*-test for paired samples differences. This test determines whether two samples are likely to have come from the same two underlying populations that have the same mean. The procedure used to apply this test was as follows. Each value of CVaR corresponding to the days in which the losses were greater than VaR was taken and its mean was obtained. Finally, the *t*-test for paired samples was applied in order to verify if the two means are statistically different.

4.4. Results

As stated in the previous section, the binomial test (Kupiec (1995)) can test the effectiveness of each model in predicting the number of losses exceeding the VaR measure. Table 4.1 presents the results for the binomial test for our four models and for the chosen 95% and 5%-VaRs.

The results show that all models did not have the null hypothesis rejected (sig.>0.05), that is the models were adequate to measure market risk for all assets. The only exception was the GARCH model used to measure 95%-VaR for the Brazilian currency, Real, against the U.S. Dollar. In this case, using the trend parameter improved the capacity of the model in estimating the market risk and the model was no longer rejected at the 95% confidence level.

Overall, however, using the appropriate EWMA model provided the best results for measuring the 95% and 5%-VaRs. By calibrating the model correctly, that is by choosing the lambda that best fits the data, one can attain superior results instead of using GARCH to forecast volatility. Additionally, using the trend parameter can improve even more the performance of the EWMA model. This was the case for the Brazilian currency, Real, and Heating Oil for 95%-VaR and the Brazilian Ibovespa index for 5%-VaR. For all the other measures that EWMA provided superior results than the GARCH model, incorporating a trend parameter did not improve its performance and even worsen its performance for 5%-VaR for Yen and S&P500. This result is expected because when we have a well calibrated model, the possibility for improvement is much lower and the trend parameter only affects considerably the VaR estimate when there is a strong trend in the market. For example, for the GARCH model, including a trend parameter improved its performance in 12 out of 16 risk measures and there was no case of performance decrease. There was much more possibility for improvement in this model than in the EWMA model. However, an investor should note that the optimal lambda for the EWMA models varied considerably across assets and confidence level. Optimal lambdas in this case varied in a range of 0.18 to 0.95. To capture trends in the market, however, most of optimal lambdas found are above 0.90. In conclusion, investors should always determine the best lambda for each VaR level and asset individually.

Table 4.1: Anticipating market risk (95% and 5%-VaRs) for different assets

	N. Obs.	GARCH		GARCH with trend			EWMA			EWMA with trend		
		Proportion	p-value	Lambda	Proportion	p-value	Lambda 1	Proportion	p-value	Lambda 2	Proportion	p-value
Panel A: 95%-VaR												
Exchange rates												
Euro	1761	5.51	0.325	0.42	5.34	0.512	0.18	5.11	0.827	0.90	5.11	0.827
Yen	1761	5.85	0.112	0.98	5.85	0.112	0.78	5.28	0.584	0.99	5.28	0.584
Real	1761	6.08	0.043	0.92	6.02	0.055	0.81	5.45	0.381	0.99	5.39	0.444
Financials												
S&P500	1758	5.46	0.381	0.81	5.40	0.443	0.45	5.06	0.870	0.86	5.06	0.870
Ibovespa	1751	5.48	0.351	0.76	5.31	0.546	0.52	5.03	0.956	0.72	5.03	0.956
Metals												
Gold	1755	5.41	0.411	0.94	5.41	0.411	0.43	5.07	0.869	0.79	5.07	0.870
Silver	1761	5.45	0.381	0.93	5.45	0.381	0.94	5.11	0.827	0.94	5.11	0.827
Energies												
Crude Oil	1751	5.08	0.869	0.99	5.03	0.956	0.65	5.03	0.956	0.94	5.03	0.956
Heating Oil	1751	5.43	0.411	0.89	5.14	0.784	0.88	5.08	0.869	0.97	5.03	0.956
Panel B: 5%-VaR												
Exchange rates												
Euro	1761	5.40	0.444	0.90	5.17	0.743	0.52	5.22	0.662	0.57	5.22	0.662
Yen	1761	5.05	0.913	0.99	5.00	1.000	0.50	5.00	1.000	0.92	5.05	0.913
Real	1761	5.79	0.126	0.78	5.45	0.381	0.59	5.00	1.000	0.91	5.00	1.000
Financials												
S&P500	1758	5.23	0.623	0.98	5.18	0.702	0.92	5.12	0.785	0.96	5.18	0.702
Ibovespa	1751	5.43	0.411	0.96	5.37	0.476	0.95	5.14	0.784	0.97	5.08	0.869
Metals												
Gold	1755	5.76	0.154	0.91	5.76	0.154	0.64	5.01	0.956	0.90	5.01	0.956
Silver	1761	5.22	0.662	0.99	5.17	0.743	0.95	5.11	0.827	0.98	5.11	0.827
Energies												
Crude Oil	1751	5.37	0.476	0.99	5.37	0.476	0.70	5.14	0.784	0.98	5.14	0.784
Heating Oil	1751	5.08	0.869	0.69	5.03	0.956	0.80	5.14	0.784	0.94	5.08	0.869

Finally, the GARCH model with trend parameter provided the best performance for the Euro against the U.S. Dollar and Heating Oil, both for 5%-VaR, and the same performance as the EWMA model for Crude Oil (95%-VaR) and the Yen against the U.S. Dollar (5%-VaR). Without the trend parameter, however, the GARCH model did not beat the other models in any of the cases.

Interestingly, for more extreme VaRs (99% and 1%) the model with the highest rate of effectiveness for the same set of assets was the EWMA model with the market trend parameter (table 4.2). The improvement provided by the trend parameter is more prominent in these cases. As the sensitivity of the VaR measure increases, the potential gain of a more accurate measure also increases. Additionally, we can note that both EWMA models (with and without trend parameter) remained with no rejection by the binomial test. At the same time, we observe that for 99% and 1%-VaRs, the GARCH model did not perform well. The binomial test rejected the null hypothesis that the predicted proportion of values exceeding VaR is equal the actual proportion observed using the model for several cases including all exchange rates, financials, and metals for 99%-VaR, and the exchange rate for the Brazilian currency, the Brazilian stock market index, and Gold for 1%-VaR.

Table 4.2: Anticipating market risk (99% and 1%-VaRs) for different assets.

		GARCH		GARCH with trend			EWMA			EWMA with trend		
	N. Obs.	Proportion	p-value	Lambda	Proportion	p-value	Lambda 1	Proportion	p-value	Lambda 2	Proportion	p-value
Panel A: 99%-VaR												
Exchange rates												
Euro	1761	1.59	0.022	0.80	1.48	0.054	0.99	1.36	0.148	0.86	1.31	0.187
Yen	1761	1.65	0.011	0.80	1.48	0.054	0.84	1.19	0.400	0.77	1.19	0.400
Real	1761	1.53	0.030	0.89	1.59	0.022	0.94	1.08	0.718	0.97	1.14	0.548
Financials												
S&P500	1758	1.65	0.011	0.28	1.59	0.022	0.85	1.31	0.187	0.85	1.19	0.400
Ibovespa	1751	1.54	0.030	0.87	1.37	0.118	0.95	1.20	0.398	0.90	1.20	0.398
Metals												
Gold	1755	1.54	0.030	0.79	1.42	0.091	0.75	1.31	0.186	0.97	1.25	0.279
Silver	1761	1.82	0.002	0.83	1.82	0.002	0.45	1.25	0.280	0.84	1.19	0.400
Energies												
Crude Oil	1751	1.09	0.717	0.92	1.09	0.717	0.49	1.09	0.717	0.86	1.09	0.717
Heating Oil	1751	1.20	0.398	0.94	1.20	0.398	0.88	1.14	0.546	0.91	1.09	0.717
Panel B: 1%-VaR												
Exchange rates												
Euro	1761	1.59	0.022	0.87	1.53	0.030	0.80	1.25	0.280	0.68	1.19	0.400
Yen	1761	1.14	0.548	0.88	1.08	0.718	0.99	1.02	0.904	0.97	1.08	0.718
Real	1761	1.65	0.011	0.94	1.59	0.022	0.44	1.25	0.280	0.70	1.14	0.548
Financials												
S&P500	1758	1.37	0.119	0.66	1.31	0.187	0.94	1.25	0.280	0.89	1.19	0.400
Ibovespa	1751	1.66	0.011	0.74	1.71	0.005	0.89	1.14	0.546	0.94	1.26	0.278
Metals												
Gold	1755	1.54	0.030	0.92	1.54	0.030	0.23	1.08	0.718	0.98	1.03	0.904
Silver	1761	1.36	0.148	0.84	1.19	0.400	0.94	1.25	0.280	0.97	1.25	0.280
Energies												
Crude Oil	1751	1.37	0.118	0.98	1.37	0.118	0.25	1.31	0.185	0.99	1.37	0.118
Heating Oil	1751	1.20	0.398	0.88	1.20	0.398	0.92	1.26	0.278	0.83	1.14	0.546

The improvement provided by the trend parameter in the GARCH model is also more evident for the tail VaRs. Incorporating the trend parameter in the VaR estimate in this case avoided rejection by the binomial test (at the 95% confidence level) for Euro (99%-VaR), Yen (99% and 1%-VaRs), Ibovespa index (99%-VaR), and Gold (99%-VaR). For the EWMA model the trend parameter increased its performance for Euro (1% and 99%-VaRs), Real (1%-VaR), S&P500 (1% and 99%-VaRs), Gold (1% and 99%-VaRs), Silver (99%-VaR), and Heating Oil (1% and 99%-VaRs) and decrease its performance for Real (99%-VaR), Yen (1%-VaR), Ibovespa (1%-VaR), and Crude Oil (1%-VaR).

Finally, as observed before, lambdas for the best EWMA model found also vary across confidence levels and assets. Lambdas for capturing market trends again are less volatile and more concentrated on large values.

The performance of the models relative to their CVaR estimation capacity is measured by two parameters. The correlation coefficient tests the capacity of the models in following tail returns oscillations. The mean paired differences test shows whether the models are consistently overestimating/underestimating CVaRs or not. The results for 95% and 5%-CVaRs for the models using the lambdas presented before are shown in table 4.3. The differences presented are the estimated CVaR minus the observed tail loss. Therefore, positive paired mean differences for 95%-CVaR (5%-CVaR) means the model is underestimating (overestimating) tail returns.

Overall, the results show superiority of the EWMA model over the GARCH model in terms of correlation between estimated tail losses and actual losses. Estimating 5% and 95%-CVaRs using the EWMA model, for most of the assets, followed more closely the variations of observed losses beyond VaR. The trend parameter did not have a major effect on the correlation coefficients of the models, although it had provided some improvement for most assets.

Interestingly, the performance of the models relative to differences between estimated CVaRs and observed tail losses show that the GARCH model is more adequate to estimate the size of the losses beyond VaR. In general, the mean of the paired differences found for this model are closer to zero and the paired *t*-test points, with higher probabilities, that they are not statistically different. We observe that for the EWMA model the mean of the paired differences found between its CVaR estimates and the observed losses beyond VaR estimates for Euro (5% and 95%-CVaRs) and S&P500 (95%-CVaR) almost reached statistical significance. Clearly, there was a tendency to the model to overestimate the market risk for these cases.

Incorporating the trend parameter to estimate losses beyond VaR did not have specific effect on the quality of the CVaR estimation of the GARCH model. It helped in some cases but also damaged in others. However, the trend parameter helped to diminish some overestimation observed in the EWMA model.

Table 4.3: Anticipating market risk (95% and 5%-CVaRs) for different assets.

	GARCH				GARCH Trend				EWMA				EWMA Trend			
	Paired Correlations		Paired Differences		Paired Correlations		Paired Differences		Paired Correlations		Paired Differences		Paired Correlations		Paired Differences	
	Correlation	Sig.	Mean (%)	Sig. (2-tailed)	Correlation	Sig.	Mean (%)	Sig. (2-tailed)	Correlation	Sig.	Mean (%)	Sig. (2-tailed)	Correlation	Sig.	Mean (%)	Sig. (2-tailed)
Panel A: 95%-CVaR																
Exchange rates																
Euro	0.711	0.000	0.002	0.929	0.662	0.000	0.023	0.442	0.709	0.000	-0.093	0.061	0.708	0.000	-0.094	0.060
Yen	0.164	0.097	-0.013	0.763	0.170	0.087	-0.014	0.743	0.560	0.000	0.007	0.862	0.561	0.000	0.006	0.877
Real	0.872	0.000	-0.036	0.424	0.873	0.000	-0.035	0.500	0.888	0.000	-0.023	0.585	0.887	0.000	-0.020	0.638
Financials																
S&P500	0.785	0.000	-0.035	0.540	0.791	0.000	-0.015	0.788	0.705	0.000	-0.126	0.109	0.724	0.000	-0.126	0.108
Ibovespa	0.411	0.000	-0.048	0.704	0.438	0.000	-0.029	0.824	0.683	0.000	-0.221	0.128	0.722	0.000	-0.209	0.148
Metals																
Gold	0.761	0.000	0.050	0.417	0.758	0.000	0.055	0.378	0.802	0.000	-0.083	0.261	0.836	0.000	-0.062	0.355
Silver	0.756	0.000	0.037	0.812	0.752	0.000	0.042	0.792	0.758	0.000	0.053	0.745	0.755	0.000	0.058	0.723
Energies																
Crude Oil	0.533	0.000	0.003	0.987	0.536	0.000	0.022	0.917	0.648	0.000	-0.172	0.349	0.642	0.000	-0.202	0.275
Heating Oil	0.719	0.000	-0.037	0.759	0.736	0.000	0.307	0.115	0.766	0.000	-0.059	0.655	0.775	0.000	-0.042	0.744
Panel B: 5%-CVaR																
Exchange rates																
Euro	0.660	0.000	0.016	0.499	0.659	0.000	0.005	0.829	0.495	0.000	0.064	0.104	0.500	0.000	0.047	0.212
Yen	0.423	0.000	-0.007	0.809	0.431	0.000	-0.011	0.718	0.651	0.000	0.047	0.220	0.636	0.000	0.043	0.246
Real	0.748	0.000	-0.021	0.777	0.773	0.000	-0.033	0.655	0.857	0.000	-0.048	0.397	0.873	0.000	-0.044	0.408
Financials																
S&P500	0.886	0.000	0.036	0.509	0.888	0.000	0.022	0.683	0.892	0.000	0.058	0.314	0.888	0.000	0.049	0.385
Ibovespa	0.572	0.000	0.005	0.956	0.564	0.000	-0.010	0.908	0.670	0.000	-0.010	0.915	0.671	0.000	-0.024	0.792
Metals																
Gold	0.458	0.000	-0.004	0.964	0.446	0.000	0.007	0.929	0.591	0.000	0.076	0.377	0.641	0.000	0.038	0.636
Silver	0.790	0.000	-0.022	0.842	0.797	0.000	-0.044	0.679	0.769	0.000	-0.031	0.780	0.770	0.000	-0.027	0.807
Energies																
Crude Oil	0.620	0.000	-0.021	0.884	0.616	0.000	-0.025	0.866	0.839	0.000	0.103	0.396	0.839	0.000	0.096	0.422
Heating Oil	0.660	0.000	-0.012	0.922	0.718	0.000	-0.016	0.889	0.764	0.000	0.031	0.788	0.772	0.000	0.017	0.884

The results of extreme CvaRs (99% and 1%) follow, in part, the results found above (table 4.4). In terms of correlation between CvaR estimates and actual tail losses, again we observe that overall the EWMA model is superior to the GARCH model, especially for 99%-CvaR. Also, incorporating the trend parameter did not show specific effect on correlations. In general, correlations remained close to the values obtained with the models without the trend parameter.

In terms of estimating the value of the tail loss/gain, we do not observe superiority of the GARCH model over the EWMA model. In fact, what we observe is high variability in terms of performance of both models. In some cases, the GARCH model performed much better than EWMA model. In others, we observe the inverse. Particularly, the GARCH model underestimated potential tail losses of Silver (0.806% on average) and both models also underestimated potential tail gains of Yen (-0.116% and -0.146% on average for the GARCH and EWMA model, respectively). In all these cases, the mean of the paired differences were statistically significant. The trend parameter also did not help avoiding these underestimations. In fact, the trend parameter contributed to the GARCH model to also underestimate tail losses of Gold and the EWMA model to also underestimate tail gains of the Brazilian currency Real. For all other assets, the trend parameter did not have major affect on the estimation of CvaR by the models.

Table 4.4: Anticipating market risk (99% and 1%-CVaRs) for different assets.

	GARCH				GARCH Trend				EWMA				EWMA Trend			
	Paired Correlations		Paired Differences		Paired Correlations		Paired Differences		Paired Correlations		Paired Differences		Paired Correlations		Paired Differences	
	Correlation	Sig.	Mean (%)	Sig. (2-tailed)	Correlation	Sig.	Mean (%)	Sig. (2-tailed)	Correlation	Sig.	Mean (%)	Sig. (2-tailed)	Correlation	Sig.	Mean (%)	Sig. (2-tailed)
Panel A: 99%-CVaR																
Exchange rates																
Euro	0.758	0.000	0.003	0.958	0.744	0.000	0.018	0.777	0.820	0.000	0.071	0.235	0.767	0.000	0.071	0.274
Yen	-0.017	0.931	-0.205	0.153	-0.010	0.963	-0.153	0.311	0.473	0.030	-0.013	0.916	0.539	0.012	-0.018	0.883
Real	0.867	0.000	-0.094	0.395	0.901	0.000	-0.200	0.245	0.940	0.000	-0.083	0.642	0.939	0.000	-0.135	0.443
Financials																
S&P500	0.686	0.000	-0.142	0.359	0.649	0.000	-0.287	0.107	0.726	0.000	0.031	0.861	0.680	0.001	0.144	0.423
Ibovespa	0.124	0.539	-0.046	0.894	0.122	0.572	0.063	0.868	0.389	0.081	0.345	0.346	0.408	0.066	0.343	0.342
Metals																
Gold	0.850	0.000	0.149	0.193	0.911	0.000	0.575	0.000	0.831	0.000	-0.050	0.650	0.855	0.000	-0.007	0.949
Silver	0.857	0.000	0.806	0.008	0.849	0.000	1.646	0.009	0.894	0.000	-0.848	0.209	0.898	0.000	-0.677	0.290
Energies																
Crude Oil	0.778	0.000	0.553	0.320	0.792	0.000	0.557	0.301	0.612	0.001	0.091	0.790	0.437	0.061	0.266	0.601
Heating Oil	0.826	0.000	0.165	0.551	0.902	0.000	0.429	0.195	0.914	0.000	0.337	0.119	0.916	0.000	0.344	0.117
Panel B: 1%-CVaR																
Exchange rates																
Euro	0.751	0.000	-0.015	0.697	0.783	0.000	-0.028	0.458	0.365	0.095	-0.017	0.827	0.414	0.062	-0.034	0.655
Yen	0.872	0.000	-0.116	0.004	0.851	0.000	-0.127	0.004	0.852	0.000	-0.146	0.001	0.835	0.000	-0.131	0.004
Real	0.781	0.000	-0.155	0.340	0.790	0.000	-0.187	0.230	0.926	0.000	-0.122	0.217	0.952	0.000	-0.243	0.003
Financials																
S&P500	0.944	0.000	0.017	0.837	0.939	0.000	-0.070	0.413	0.942	0.000	0.049	0.615	0.942	0.000	0.019	0.845
Ibovespa	0.435	0.018	0.150	0.512	0.437	0.016	0.139	0.532	0.643	0.002	-0.124	0.643	0.599	0.003	-0.003	0.990
Metals																
Gold	0.585	0.001	-0.228	0.250	0.594	0.001	-0.231	0.240	0.944	0.000	0.007	0.944	0.946	0.000	-0.001	0.990
Silver	0.779	0.000	-0.125	0.516	0.784	0.000	-0.194	0.362	0.787	0.000	-0.114	0.568	0.788	0.000	-0.101	0.613
Energies																
Crude Oil	0.696	0.000	0.041	0.907	0.689	0.000	0.019	0.958	0.618	0.002	0.050	0.888	0.612	0.001	0.091	0.790
Heating Oil	0.746	0.000	-0.161	0.562	0.744	0.000	-0.116	0.678	0.843	0.000	-0.233	0.340	0.865	0.000	-0.241	0.367

Summarizing, the results of this study indicate a superiority of the volatility weighted historical simulation using the EWMA model instead of the GARCH model to forecast volatility to estimate both non-extreme VaRs (5% and 95%) and extreme VaRs (1% and 99%). Additionally, it is shown that using the trend parameter suggested in this study can improve the performance of these models in estimating VaRs, especially the extreme ones. In terms of CVaR estimation the results are ambiguous. There is no prevalence of one model over the other. Also, the contribution of the trend parameter to these models in estimating CVaRs is not evident.

4.5. Conclusion

This study aimed at verifying empirically the applicability of adding a new parameter (market trend) into the volatility-weighted historical simulation approach to measure market risk (VaR and CVaR) for different groups of assets (exchange rates, financials, metals, and energies). The period used for our analysis goes from the first day of trade of 2001 to the last day of trade of 2007. Volatilities are forecasted by using EWMA and GARCH models. Validation tests used are the binomial test, applied over the failure rates (proportion of times VaR is exceeded), and the *t*-test for paired samples differences, used to evaluate the effectiveness of the CVaR measures in predicting the value of the losses when these are greater than VaR estimates.

The results show that adding the market trend parameter can improve VaR estimates, especially for the cases of extreme VaRs (99% and 1%). Also, we observe that overall using a well calibrated EWMA model to forecast volatility provides better VaR estimates for the volatility-weighted historical simulation approach than using the GARCH model. This pattern is also more prominent for the cases of more extreme VaRs. We observe that for 99% and 1% VaRs, the GARCH model did not perform well, being rejected by the binomial test several times.

In terms of CVaR estimates there is no clear superiority of a model over the other. Higher correlations between CVaR estimates and actual tail returns is observed for the volatility weighted historical simulation approach using the EWMA model to forecast volatility but lower paired differences mean is observed for the GARCH model for 5% and 95%-CVaRs. In terms of extreme CVaR estimation (1% and 99%) the results are ambiguous. There is no prevalence of one model over the other. Also, the contribution of the trend parameter to these models in estimating CVaRs is not evident.

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Appendix A: Derivation of the EWMA model[28]

Using the most recent m observations of daily returns, an unbiased estimate of the variance rate per day, σ_t^2 , is:

$$\sigma_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_{t-i} - \bar{r})^2 \qquad \text{(A.1)}$$

Where $\bar{r}$ is the mean of daily returns.

When dealing with daily returns the mean will be very low, and we can approximate it as been zero. Also, if n is a large number, we can replace the n-1 term by n. In fact, according to Figlewsky (1994), these simplifications often make very little difference to the variance estimates, and usually reduce their standard errors. We then have the following equation for the volatility estimate:

$$\sigma_t^2 = \frac{1}{n}\sum_{i=1}^{n} r_{t-i}^2 \qquad \text{(A.2)}$$

It is easy to note that the equation above gives equal weights to all r_i^2 's. However, given that the objective is to monitor the current level of volatility, it is more plausible to give greater weight to more recent observations and less weight to more distant ones. For that we can use a moving average scheme with declining weights. This approach fits better the fact that volatility tends to change over time. So, the estimate becomes:

$$\sigma_t^2 = \sum_{i=1}^{n} \alpha_i r_{t-i}^2 \qquad \text{(A.3)}$$

where the variable α_i is the amount of weight given to the observation i days ago. It declines as i gets larger, $\alpha_i < \alpha_j$ when $i > j$, and sum to 1.

One way to deal with the fact that volatilities vary over time is to use the exponentially weighted moving average (EWMA) model. In this approach the weights, α_i , decrease exponentially as we move back trough time. Specifically, $\alpha_{i+1} = \lambda \alpha_i$, where λ is a constant between zero and one. The formula for updating volatility becomes:

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[28] See also Dowd (2002).

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{n}\lambda^{i-1}r_{t-i}^2 + \lambda^n\sigma_0^2 \tag{A.4}$$

For a large n, the term $\lambda^n\sigma_0^2$ is sufficiently small to be ignored. Lagging the equation above by one period, and multiplying throughout by λ, we get:

$$\lambda\sigma_{t-1}^2 \approx \lambda(1-\lambda)\sum_{i=1}^{n}\lambda^{i-1}r_{t-i-1}^2 \tag{A.5}$$

and, therefore,

$$\lambda\sigma_{t-1}^2 = (1-\lambda)\sum_{i=1}^{n}\lambda^{i}r_{t-i-1}^2 \tag{A.6}$$

Subtracting equation (13) from equation (11) and rearranging gives:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 - (1-\lambda)\lambda^n r_{t-n-1}^2 \tag{A.7}$$

Finally, the estimates of the volatility using the EWMA model becomes:

$$\sigma_t^2 \approx \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 \tag{A.8}$$

Where the estimate, σ_t, of the volatility for day t (made at the end of day t-1) is obtained from σ_{t-1} (the estimate from one day ago of the volatility for day t-1) and r_{t-1} (the most recent observation on changes in the daily return).