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The implications of variable or constant expansion rates in invasive weed infestations

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Data on the spread of invasive weeds into arid western lands are used to evaluate the environmental and economic importance of controlling invasive weed infestations early. Variable rate and constant rate infestation expansion paths are estimated. The implications of variable vs. constant infestation growth rates for projecting both biophysical and economic effects are illustrated. The projections derived from both constant and variable growth rate expansion paths support the contention that it is expedient to control new infestations early.

Key words: Invasive weeds, infestation expansion paths, eradication and revegetation costs, present value.

Numerous invasive and noxious weeds currently infest or threaten western ecosystems. Many are well established; others are either in the process of becoming established or are expected to invade. It is critically important, for both economic and environmental reasons, to attack new infestations of invasive and noxious weeds vigorously and early, before they have an opportunity to become well established. However, in the competition for scarce resources, these claims must contend with countless other urgent demands. Therefore, it is important to provide managers and policy makers with the best possible evidence on the need to deal promptly and adequately with the invasive weed threat facing western ecosystems.

In biological systems, economic effects derive from biophysical effects. Projecting the economic effects incurred by delaying eradication of invasive weed infestations must begin with projection of an infestation expansion path. This investigation found neither documented nor consistent evidence on the establishment and growth patterns of invasive weed infestations, particularly in arid western ecosystems. Nonetheless, it is imperative to do the best possible job with the limited data that are available; consequently, this results in an analytical approach based upon assumptions.

Cousens and Mortimer (1995) discussed the theoretical stages of weed infestation development. They explained why rates of expansion should be expected to vary throughout a process of infestation development and concluded that a lack of sufficient data is the major factor limiting better analysis and estimation of infestation growth patterns. Forcella (1985) analyzed the rate at which alien species had spread through the northwestern U.S. by tabulating the annual increases in the numbers of counties infested by each species. This approach does not lend itself to the quantification of either biophysical or economic effects, but it demonstrates the urgency of what he terms "weed epidemiology."

In discussing the expansion of noxious range weed infestations, Callihan and Evans (1991) observed that noxious weed infestations for which documentation is available appear to be expanding at average rates of from 3 to 60% per year. They attribute this variation to niche availability. They also point out that western rangeland has such low residual grass populations that establishing protective vegetation following weed eradication is an essential consideration when estimating economic effects. Sheley et al. (1996) also emphasized the role of establishing and maintaining vigorous plant communities as a primary defense against invasive weeds.

Other literature reports expansion rates that vary, even within a single species, but no attempt is made to quantify variable rate expansion paths. Instead, estimation is limited to calculating constant, average rates of expansion over the duration of the observations. For example, expansion rates for 14 species of noxious weeds ranged between 8 and 24% and averaged 13.4% (Asher 1985). Similar variation occurred in the expansion rates of knapweed (*Centaurea* spp.) infestations (Roche et al. 1994).

Methods

Database

A database of 35 observations on the expansion of invasive weed infestations in the Great Basin or nearby states with similarly arid environments was compiled by soliciting information from western weed and range specialists. Data were sought for those invasive species that have been identified as of general concern throughout the Great Basin.

These observations were reported in two ways. Twenty observations report an estimate of the infestation size at some initial point in time and a subsequent estimate of the total acres infested at some later date. Another group of 15 observations contains a date when the weed was first reported in a region, an intervening period of years, and a subsequent estimate of the total number of infested acres in the region. That these 15 observations do not specify a size

Table 1. Observations on initial and final weed infestation sizes. Time refers to the number of years between the initial and final observations.

Weed species	Initial size	Final size	Time
	ha	ha	yr
Common crupina (*Crupina vulgaris* Cass. CJNVU)	Unknown	9,308	16
	239	24,282	30
Dyers woad (*Isatis tinctoria* L. ISATI)	4,856	60,704	8
	Unknown	9,713	51
	14.2	718	16
Diffuse knapweed (*Centaurea diffusa* Lam. CENDI)	61	526	8
Spotted knapweed (*Centaurea maculosa* L. CENMA)	Unknown	1,821,125	76
	121	648	8
	Unknown	203,561	44
	Unknown	688	12
	152	1,760	16
Squareose knapweed (*Centaurea virgata* Lam. CENVS)	0.4	60,704	46
	16,188	40,496	8
Yellow star thistle (*Centaurea solstitialis* L. CENSO)	Unknown	121,408	30
	405	56,657	42
	404,695	4,046,945	19
	Unknown	74,868	32
	Unknown	848	25
	16.2	202,347	42
Canada thistle (*Cirsium arvense* Scop. CIRAR)	Unknown	930,797	100
Musk thistle (*Carduus nutans* L. CRUNU)	10,927	173,209	8
	162	1,862	16
Scotch thistle (*Onopordum acanthium* L. ONRAC)	6,070	14,164	8
Leafy spurge (*Euphorbia esula* L. EPHES)	Unknown	404,695	30
	Unknown	617	52
	263	1,255	16
Medussahead (*Taeniatherum caput-medusae* L. ELYCM)	Unknown	1,821,125	52
	Unknown	607	8
	Unknown	1,821,125	46
Rush skeletonweed (*Chondrilla juncea* L. CHOJU)	16.2	1,618,778	33
	Unknown	1,416,431	23
	202	202,347	36
	21,246	50,992	16
Tall whitetop (*Lepidium latifolium* L. LEPPE)	2,833	6,880	8
Purple loosetrife (*Lythrum salicara* L. LYSTA)	0.81	20,235	16

for the infestations at the time of their first report causes considerable difficulty in the subsequent analysis. The differences in the average time spans of the observations contained in these two groups are also important. The 20 observations, which report an initial size for the infestation, span on average 20 yr and range from 8 to 46 yr. By contrast, the observations in the group of 15, which do not specify an initial infestation size, span an average of 39.8 yr and range from 8 to 100 yr. Six of these observations span > 46 yr. Thus, the 20 observations, which stipulate a starting size for the infestations being reported, cover observation periods only about one-half the length of those covered by the other 15 observations. The complete data set of 35 observations is presented in Table 1.

These observations stem from a variety of sources and probably differ in accuracy. Therefore, it is necessary to make some assumptions about the data set as a whole. First, it is assumed that, taken together, these observations create an unbiased sample, i.e., the infestation size estimates are equally likely to be either high or low. Next, it is assumed that no effective control or management programs substantially influenced the progressions of these infestations during the period of the observations. Finally, it is assumed that local climatic and environmental conditions during the observation periods were typical for the region.

Data Analysis

Infestation expansion rates are certain to vary by species, climate, soil conditions, traffic patterns, and numerous other variables. Nonetheless, the purposes and data limitations of this analysis dictate estimation of generalized expansion paths that ignore many important variables and vary only with the initial infestation size and subsequent duration of expansion. This approach permits estimation of the adverse effects brought about by delaying eradication and control efforts.

There are two ways in which total infestations expand. Lateral expansion takes place at the perimeters of existing infestations. Simultaneously, new infestations may occur at distant locations. These new infestations need not originate from sources within the region. They may originate from propagules brought in from foreign sources, as did initial infestations. This is particularly likely if the activities that led to the original introductions are continued. Thus, a total rate of infestation expansion over a period of years is a compounding of these two methods of increase.

An applicable formula for compounding rates of growth that remains constant throughout the growth period is routinely employed in many economic contexts:

$$E = (1 + g)^t B \quad [1]$$

where E is the infestation size following expansion, B is the initial size, g is some constant rate of growth, and t is the number of compounding periods—in this case, years. Equation 1 is transformed for estimation by first dividing B into E, then taking the natural logarithm of both sides of the resulting equation.

Because rates of expansion are expected to slow over time, inclusion of observations with longer durations can be expected to reduce the overall average growth rate. By eliminating the 15 observations that lack starting sizes, a data set of 20 observations with an average time span of 20 yr is left. This time span coincides more closely with the period of early infestation dynamics, which is the focus of this research. Furthermore, because the infestation starting sizes are available in this data subset, the estimation can be accomplished using traditional ordinary least squares methods.

Tests revealed that the 20 shorter time span observations used above are insufficient to estimate significant curvature in the rate of expansion. Curvature in this usage is synonymous with a growth rate that is not constant but varies through time. Similar tests found the sample of 15 observations with missing starting size values to be robust in indicating significant curvature throughout a wide variety of starting size assumptions. This difference is attributed to the longer durations spanned by the observations in the smaller subset. Therefore, it is necessary to include both subsets of data when estimating a variable expansion rate equation. Using both subsets of data necessitates finding a plausible method for estimating starting sizes for those observations that lack them. In estimating these missing starting infestation sizes, it is important to avoid, as far as possible, statistical pitfalls, such as simultaneous equation bias. For this reason, maximum likelihood was used to estimate the following equation:

$$\begin{aligned} I = \beta_1 T + \beta_2 T^2 + (1 - d)(1 + \beta_3 T)L \\ + d(1 + \beta_3 T)\{(T^{-\beta_4})I + u\} + e \end{aligned} \quad [2]$$

The dummy variable d is set at 1 for observations in which the starting sizes are missing and at zero otherwise. When $d = 1$, the portion of the equation in brackets estimates beginning size as a variable proportion of the ending infestation size. This variable proportion is parameterized as a function of time and provides for a random error term, u, which enters the beginning size construct independently of the expansion rate error term, e. The variable I is the natural logarithm of the ending infestation size, and L is the natural logarithm of initial infestation size, both specified in hectares. T is the time variable representing the duration of infestation expansion specified in years. The inclusion of I on the right-hand side of the equation is dealt with by entering the Jacobian of the transformation into the likelihood algorithm. Other variable and parameter names are consistent with mathematical convention and familiar abbreviations.

TABLE 2. Maximum likelihood estimation of a nonlinear rate of expansion.

Parameter	Estimate	Standard error	t statistic
β_1	0.49003	0.048522	10.099
β_2	−0.0037979	0.0011609	−3.2715
β_3	−0.029239	0.0028325	−10.323
β_4	0.30246	0.16229	1.8637
σ_e	1.5962	0.25321	6.3037
σ_e	5.569	1.814	3.0744

Results and Discussion

Regression Results

Ordinary least squares regression of Equation 1 provides an estimated constant average expansion rate of 23.7% yr^{-1}. The regression statistics show good significance, with a t statistic of 9.4 on the parameter estimate and an adjusted R^2 of 0.58. This R^2 indicates that 58% of the variation in the data is explained, a reasonably good fit for a function that averages across a diversity of species and other important variables.

The results from the maximum likelihood estimation of Equation 2 show statistically significant curvature in the rate of infestation expansion (Table 2). The negative signs on β_2 and β_3 are consistent with an expansion path that slows, in percentage terms, as time and starting infestation size increase. The positive sign on β_4 is similarly consistent, because this parameter enters the equation with a negative sign. The projected rates of infestation expansion for the early years of small infestation sizes are similar to the 60% expansion rates found in the literature (Callihan and Evans 1991; Roche et al. 1994). Unrealistically, however, this equation has so much curvature that it projects expansion rates will become negative, and infestations begin to decrease in size at approximately 1 million ha. While negative expansion rates in later years are troubling, it is not uncommon for polynomial functions to take unrealistic paths toward the outer limits of the range of data used to estimate them. However, one could reasonably expect approximate consistency in expansion rates projected for any given size of infestation. This equation produces different annual expansion rate projections for infestations of approximately the same size, when different starting size assumptions are used. Therefore, these results provide statistically significant evidence of expansion rates that slow as an infestation progresses; however, better data through research of expansion of weed infestations are needed for more accurate estimating. Figure 1 contains estimates from selected points along four expansion paths that were projected for starting infestation sizes of 0.1, 1, 10, and 100 ha.

After 20 yr of expansion, a constant 23.7% annual rate of growth applied to an original 10-ha infestation results in infestation of 704 ha. By contrast, the variable rate expansion path previously estimated with Equation 2 projects a 10-ha infestation growing to 6,054 ha in 20 yr. The difference between these estimates is attributable to the much higher rates of expansion projected for the early years of infestation expansion.

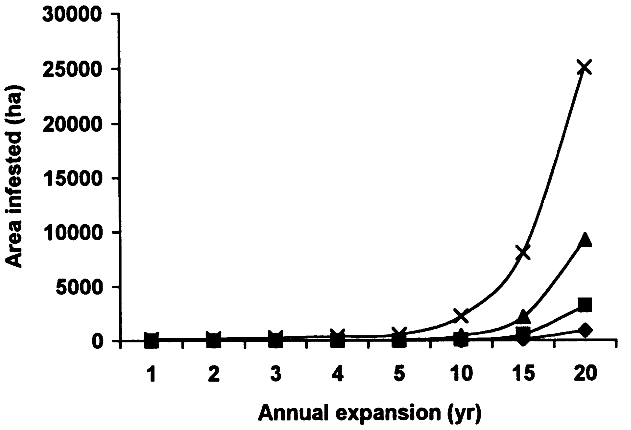

FIGURE 1. Projected weed expansion paths following initial infestations of 0.1 (♦), 1 (■), 10 (▲), and 100 (x) ha. Data were fitted to Equation 2 (see text for details).

Economic Projections

Comparing the costs of early eradication to the costs incurred by delaying eradication involves the comparison of sums and flows of money that are projected to occur at different times. Because of the time value of money, monetary amounts must always be discounted to a common point in time before being compared. The method used here is referred to as present value analysis. A nominal 8% interest rate yr^{-1} is used throughout this work to discount future monetary amounts to a common time point. This common time point may be thought of as coincident to the earliest point at which an infestation is identified and eradication could conceivably begin.

Eradication can seldom be obtained through a single treatment. Throughout the following analysis, treatments to eradicate infestations will be assumed 75% effective. Therefore, in the year following a treatment, acreage equal to 25% of the previous year's acreage will be assumed to require additional treatment. This declining series of treatments is assumed to continue until the subsequent year's area drops to 4 ha or less. Within a 20-yr projection horizon, a declining series of treatments for infestations of the sizes projected here can take between 1 and 7 yr to reduce an infestation to a maintenance level of 4 ha or less. Following both early and delayed eradication, it is assumed that an annual maintenance regimen of spot treatments totaling 4 ha will be necessary to control scattered regrowth and new infestations that occur. Because, in the years following a delayed eradication, this maintenance requirement would apply equally to both early and delayed scenarios, the costs for these maintenance treatments from the time that delayed eradication is completed onward into the infinite future can be ignored in the comparisons. However, when calculating the costs of early eradication, this maintenance requirement can not be ignored for those years following early eradication and lasting until delayed eradication is completed.

Following eradication of small infestations, it is usually possible to avoid revegetation expenses by relying on nearby native plants to provide a sufficient seed source. Data on this are both scanty and certain to vary with innumerable factors peculiar to each weed species and site. For this evaluation, it is assumed that for infestations of ≥ 16ha, one-half the total infested area will be in monocultures of sufficient size to require revegetation. A revegetation cost of \$175 ha^{-1} is posited.

TABLE 3. Eradication costs of weed infestations following immediate treatment and subsequent maintenance (A) and delayed treatment based either on a constant (B) or variable (C) rate expansion path.

	Eradication cost		
Delay	A	B	C
yr			
		10 ha[a]	
5	$2,337	$3,914	$9,125
10	$3,316	$7,895	$34,062
15	$3,981	$15,563	$104,654
20	$4,435	$30,854	$265,737
		100 ha[a]	
5	$21,753	$40,317	$65,480
10	$22,731	$79,384	$173,843
15	$23,397	$156,678	$381,233
20	$23,850	$308,886	$691,338
		1,000 ha[a]	
5	$205,710	$403,123	$468,269
10	$206,688	$794,983	$886,810
15	$207,354	$1,567,152	$1,388,810
20	$207,807	$3,089,157	$1,798,611

[a] Indicates initial weed infestation.

Herbicide treatment costs of \$90 ha^{-1} are posited, as a compromise between the chemical costs for chlorsulfuron and the dimethylamine salt of 2,4-D. There is no intent that these cost estimates should be viewed as realistic forecasts of what true costs might be. These costs are intentionally positioned at the low end of reported costs. The use of low estimates is intended to focus attention on the effect of compounding when eradication and revegetation projects are delayed. The magnitude of expenditures is not critical for purposes of contrasting immediate vs. delayed control scenarios.

The above assumptions are used to project comparable costs at different points along the previously estimated expansion paths. When calculating the immediate eradication scenarios, it is assumed necessary to follow eradication with a program of spot treatments, which annually average 4 ha. The cost of this maintenance program throughout the period between immediate and delayed eradication is included in calculating the immediate eradication cost estimates. As previously noted, following delayed eradication, both scenarios would equally require these maintenance efforts, so maintenance costs need not be included for purposes of comparison.

Table 3 contains estimated total present values of the projected eradication costs for three scenarios. The first projects immediate eradication and maintaining a program of spot treatments over the specified number of years of delay to which it is to be compared. The second projects delayed eradication, assuming a constant rate expansion path as estimated above. The third projects delayed eradication, assuming the variable rate expansion path as previously estimated. As previously noted, an 8% interest rate is used to discount all monetary amounts to a common time point, coincident to immediate eradication.

The projected expansion paths indicate that effects on resources, ecosystems, and biodiversity accelerate rapidly when measures to eradicate an infestation are delayed. When estimated infestation expansion paths are used to generate eradication and revegetation cost estimates, the present value of projected costs from failing to eradicate new infestations early also accelerates quickly.

This analysis supports the contention that an early and vigorous approach to the eradication of new invasive weed infestations is expedient, for both environmental and economic reasons. It also supports policy recommendations that we implement programs to manage large, well-established infestations in ways that can minimize enormous annual increases in infested acreage that will otherwise occur. The analysis demonstrates the need for better data and further research.

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